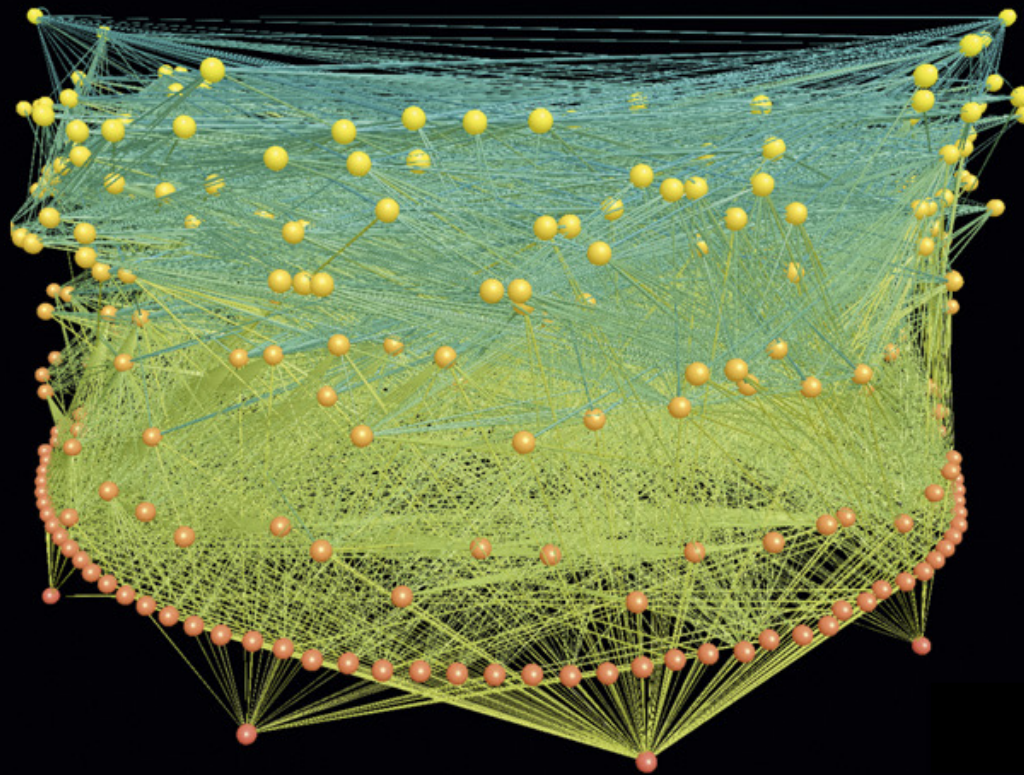


Trait-based Eco-evolutionary Theory

Christopher A Klausmeier

Kellogg Biological Station,
Departments of Plant Biology & Integrative Biology
Michigan State University

Many interacting components

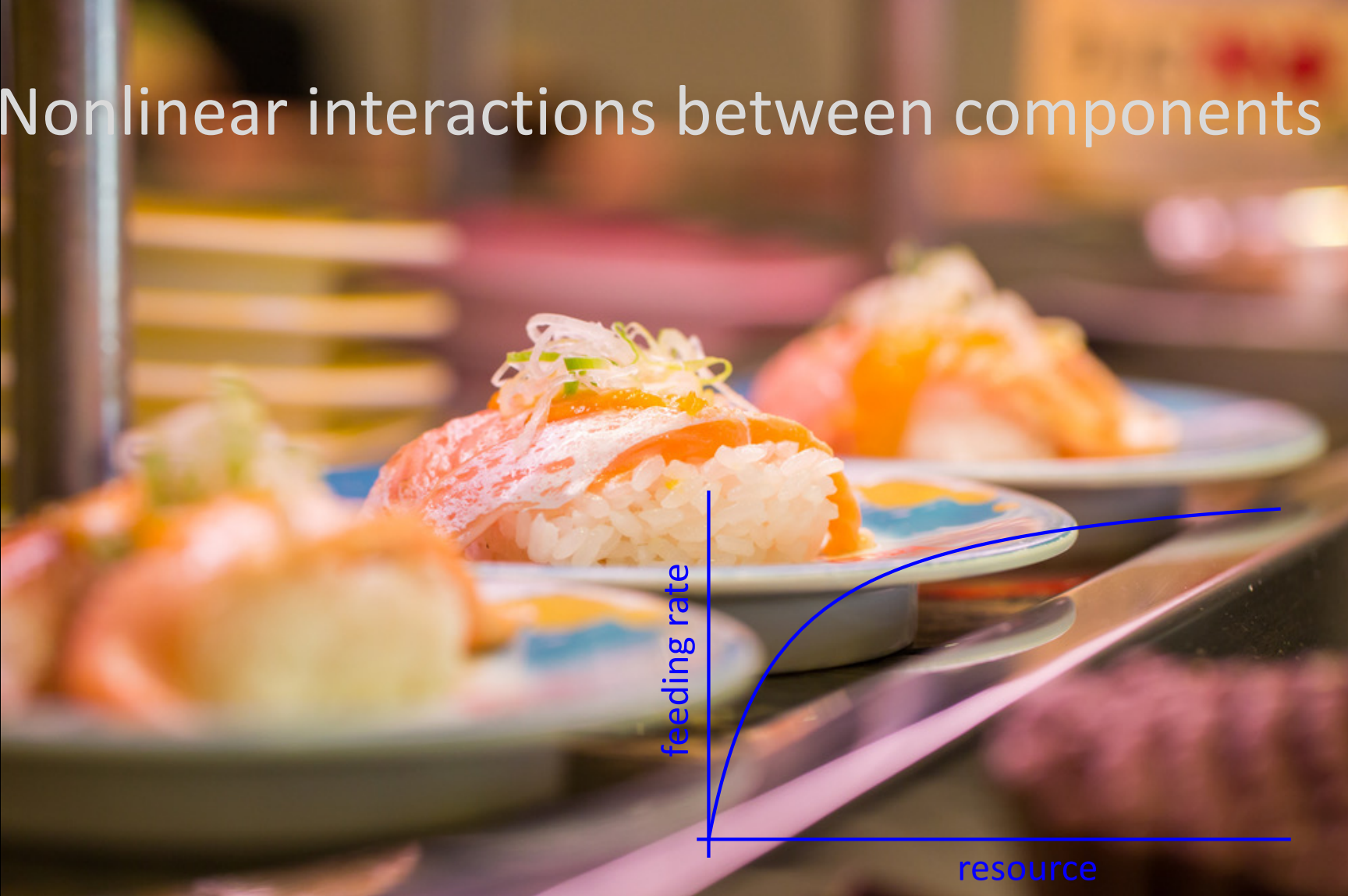


Weddell Sea food web from Jacob *et al.* 2011

Heterogeneity between & within components



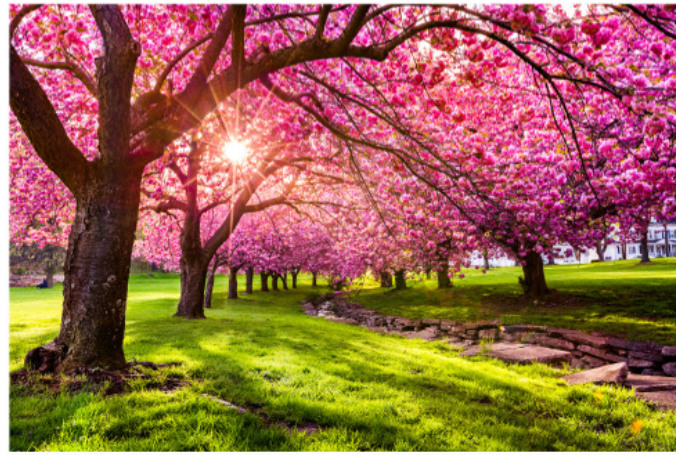
Nonlinear interactions between components



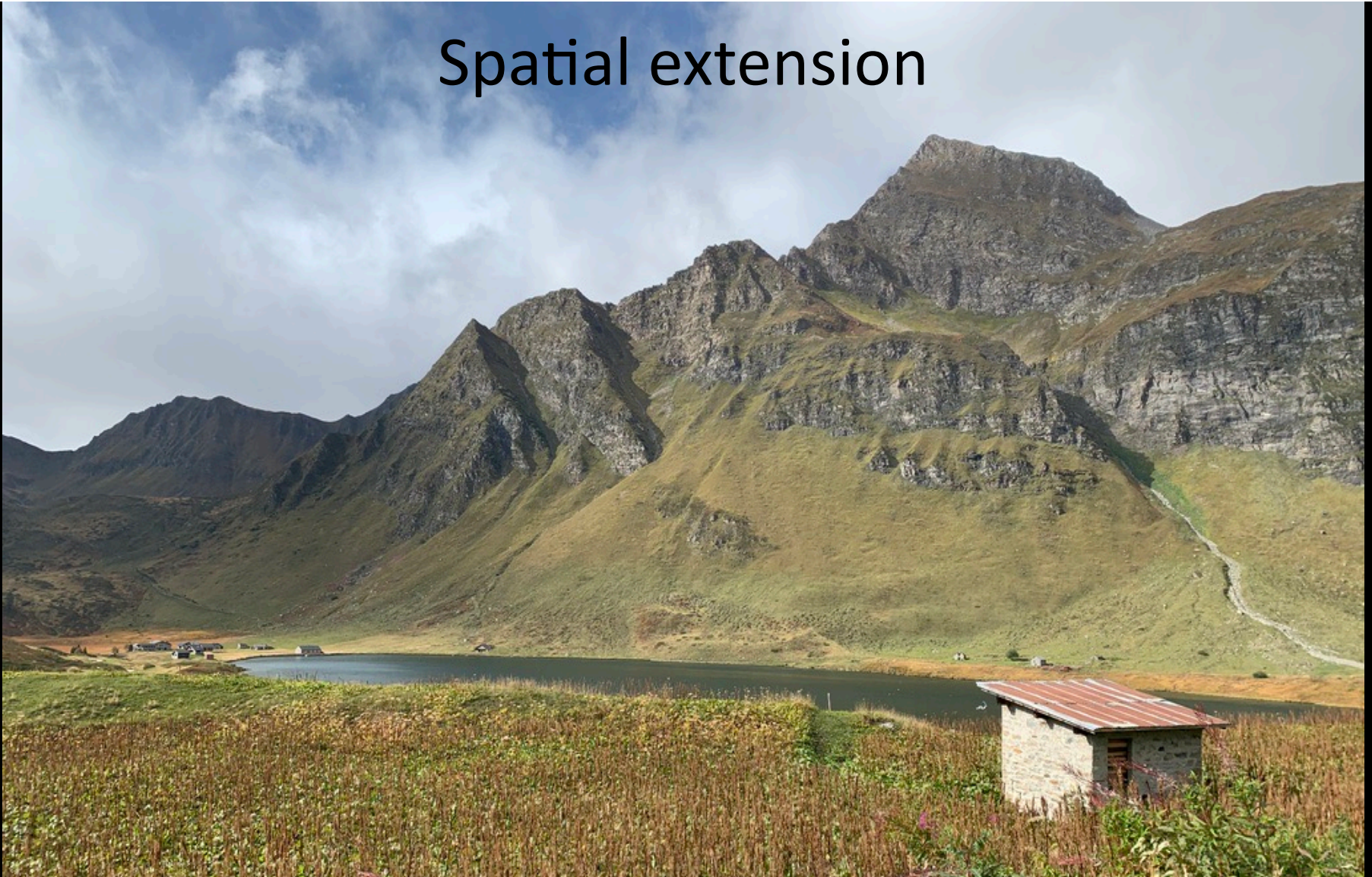
feeding rate

resource

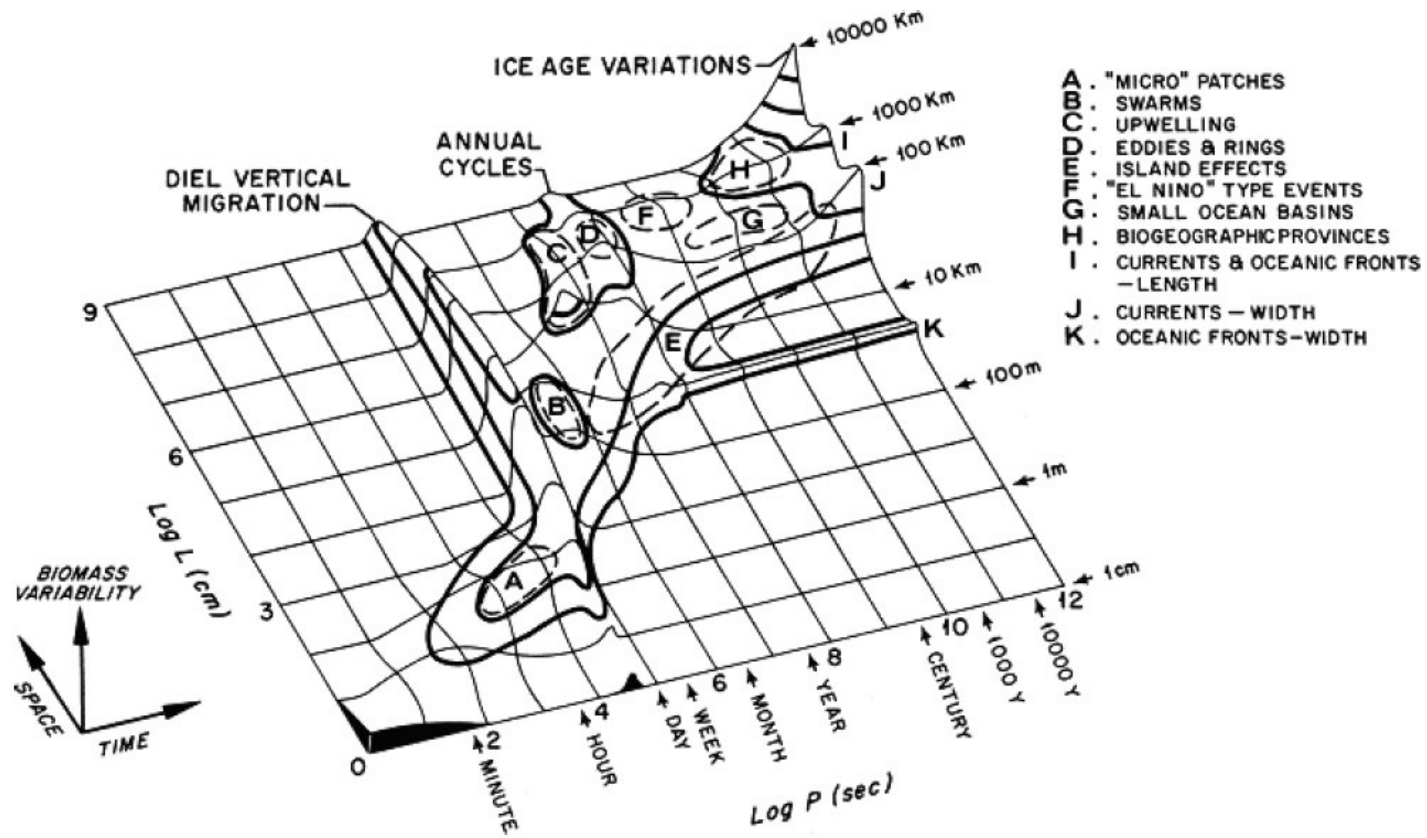
Temporal variability



Spatial extension



Multiple scales








Stommel diagram from Haury *et al.* 1978

Communities as Complex Systems

Many interacting components

- ✓ Heterogeneity between & within components
- ✓ Nonlinear interactions
- ✓ Temporal variability
- ✓ Spatial extension
- ✓ Multiple scales

Communities as Complex Systems

-  Emergent properties
-  Indirect effects, network effects
-  Negative and positive feedback loops
-  Tipping points, hysteresis, alternative stable states
-  Self-organization

Predicting Ecosystem Responses to Environmental Change



Predicting Ecosystem Responses to Environmental Change

- Direct effects + indirect effects mediated by community structure
- Ecosystem function depends on environment E , population size N , traits, x — $F(E, N(E), x(E))$

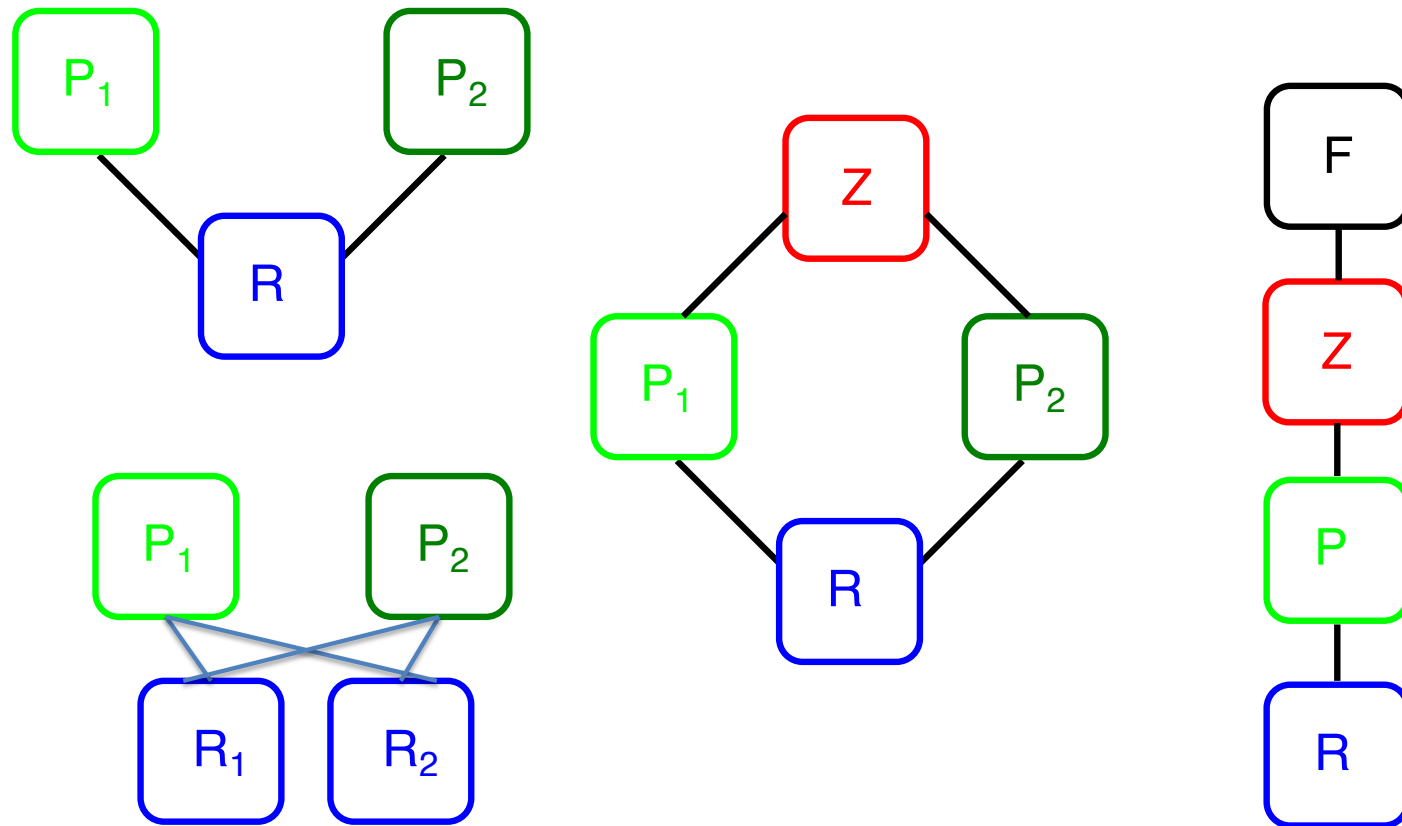
$$\frac{dF}{dE} = \frac{\partial F}{\partial E} + \frac{\partial F}{\partial N} \frac{dN}{dE} + \frac{\partial F}{\partial x} \frac{dx}{dE}$$

- Trait change through evolution or community reorganization could buffer or exacerbate response to change

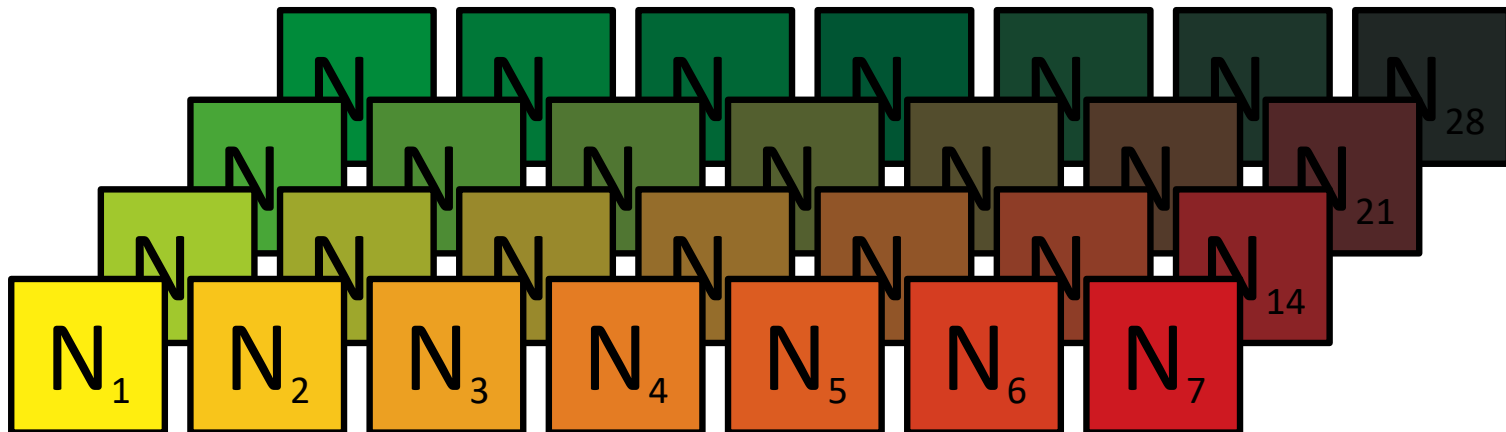
Outline

- I. Ecological communities as complex systems
- II. Introduction to trait-based eco-evolutionary theory (adaptive dynamics)
- III. Evolutionary rescue (quantitative genetics)
- IV. A general framework combining intra- and interspecific trait variation (multi-species moment methods)

Traditional Community Ecology Models

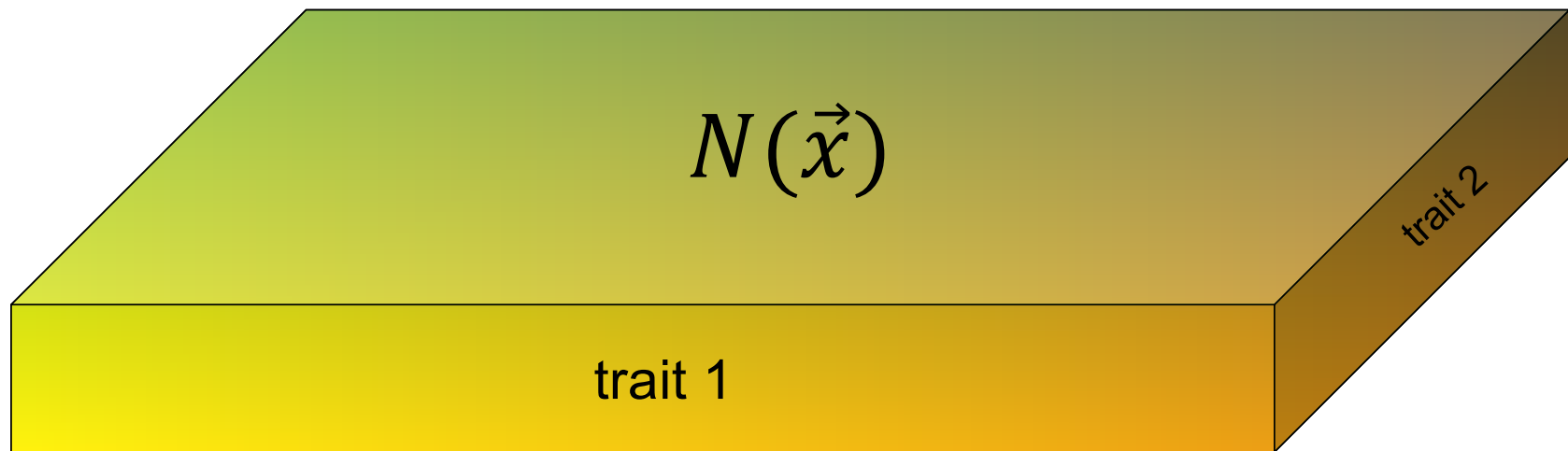


Problem: How to Incorporate Biodiversity?



- indirect effects & complex dynamics possible
- # of interaction coefficients scales with number of species \mathcal{N} as \mathcal{N}^2

Trait-Based Eco-Evolutionary Theory



- parameterize species by their functional traits
- conceptual unification of ecology & evolution

Trait-Based Eco-Evolutionary Theory

When we turn to biological systems, composed of a number of “kindred-groups,” we observe an analogous state of affairs. In general the individuals comprised within a kindred-group are not all precisely similar. Thus, expressing the matter analytically, out of a total N_1 of individuals of some group A_1 , a certain fraction

$$N_1 C_1 (p, q, r, \dots) dp dq dr \dots$$

will have the values of certain characteristic features P, Q, R, \dots comprised between the limits

$$\begin{aligned} p \text{ and } (p + dp) \\ q \text{ and } (q + dq) \\ r \text{ and } (r + dr) \\ \dots \end{aligned}$$

A similar statement holds for each of the other groups A_2, A_3, \dots

As time goes on both the values of N_1, N_2, \dots will in general change, and also the form of the frequency functions C_1, C_2, \dots . In other words, the matter of the system undergoes a change in distribution: (1) among the several kindred-groups; (2) among the several types of individuals of which each group is composed. The former change may be spoken of as “Inter-Group Evolution,” the latter as “Intra-Group Evolution.”⁵

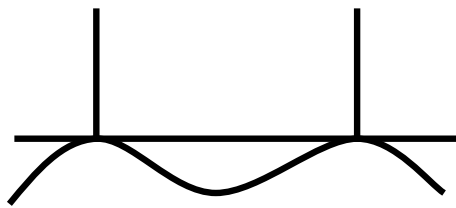
It is intra-group evolution, the change in time of the character of a species, with the possibility of the origin of a new species as its outcome, which has hitherto mainly engaged the attention of the biologist.

We, on the contrary, will here turn our attention chiefly to certain aspects of inter-group evolution.

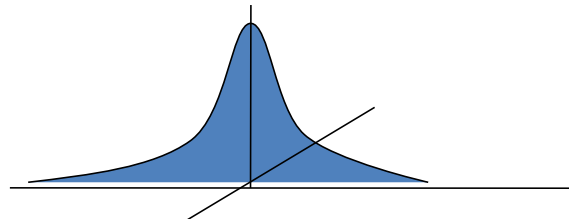
(Lotka 1912 *J Wash Acad Sci*)



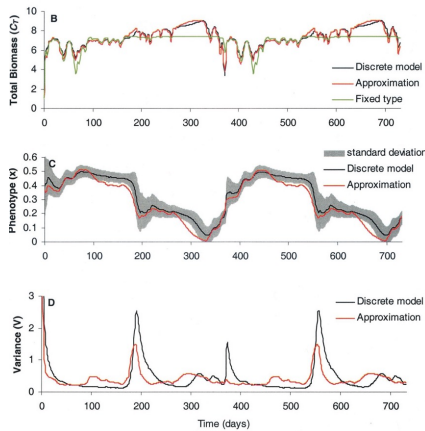
Trait-Based Eco-Evolutionary Theory



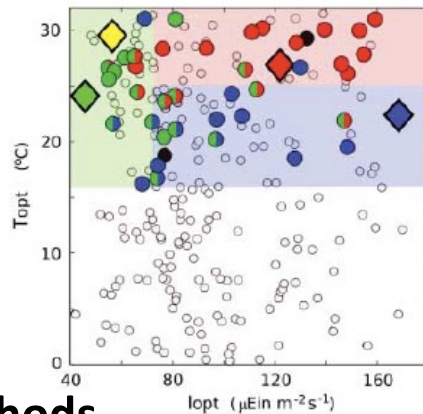
ESS Maximum Approach
(Brown, Vincent)



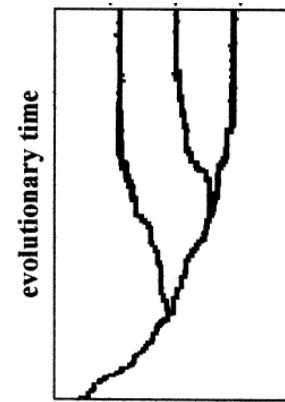
Quantitative Genetics
(Lande, Abrams)



Ecological Moment Methods
(Wirtz, Norberg,
Webb, Savage, Bruggeman)



Monte Carlo
(Follows et al.)

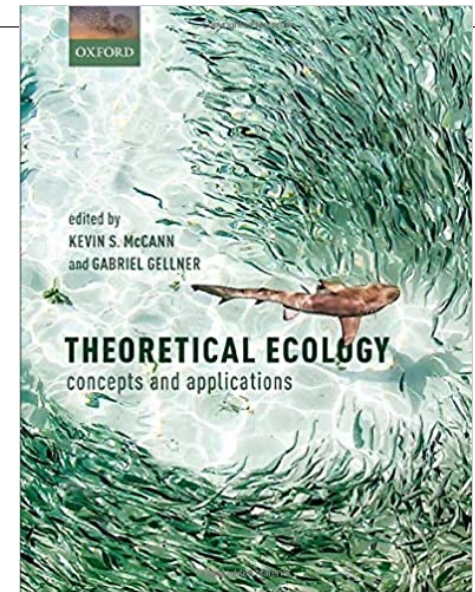


Adaptive Dynamics
(Geritz, Metz, Dieckmann, Law)

Reviewed in:
CHAPTER 11

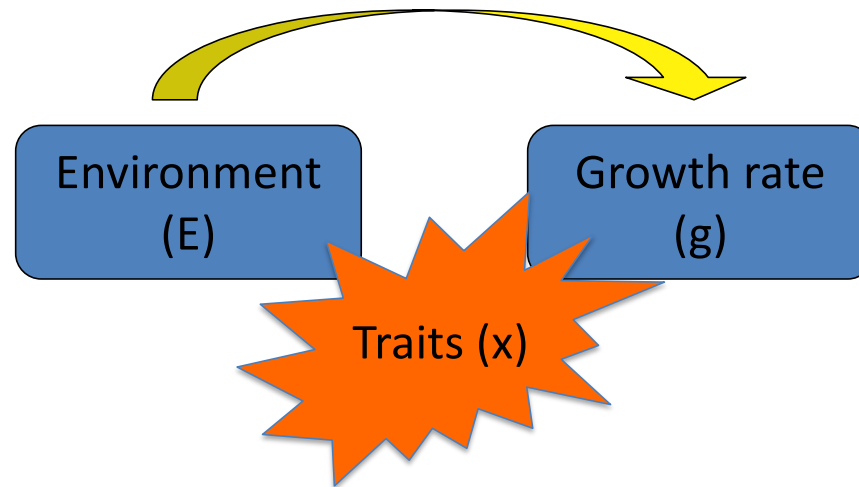
Trait-based ecological and eco-evolutionary theory

Christopher A. Klausmeier, Colin T. Kremer, and Thomas Koffel



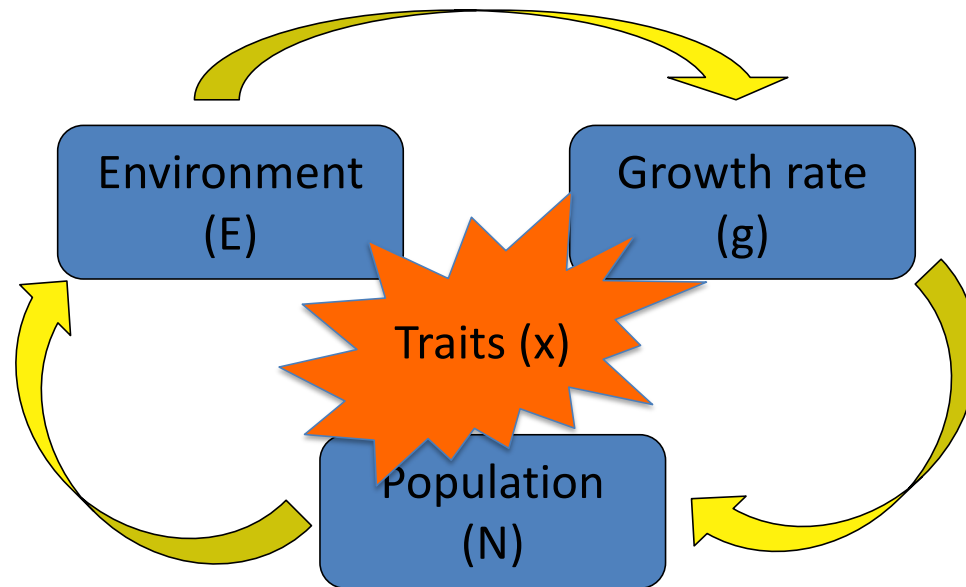
(in McCann & Gellner, eds., 2020)

Optimization



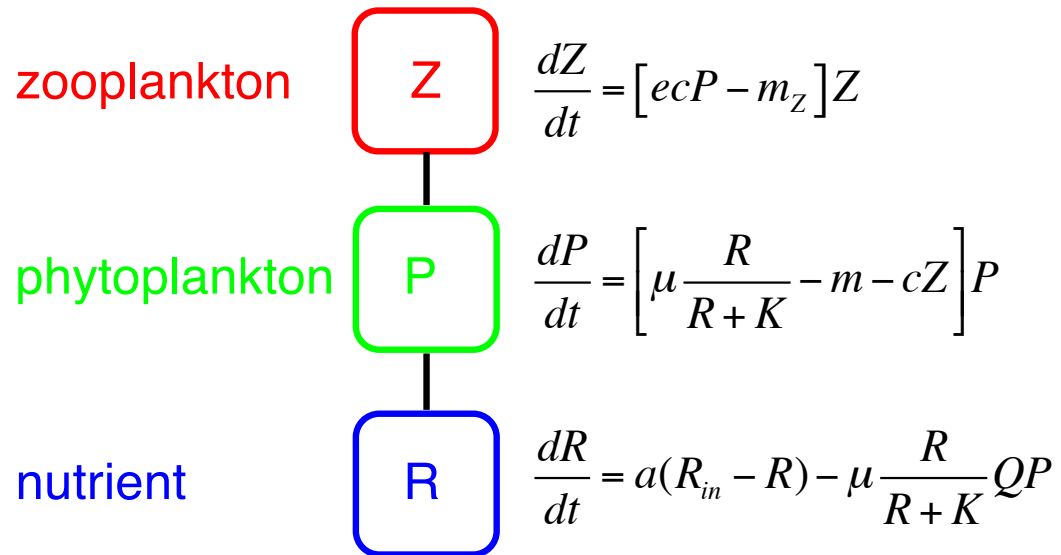
$$\frac{dN_i}{dt} = g(x_i; E)N_i$$

Game Theoretical Approach

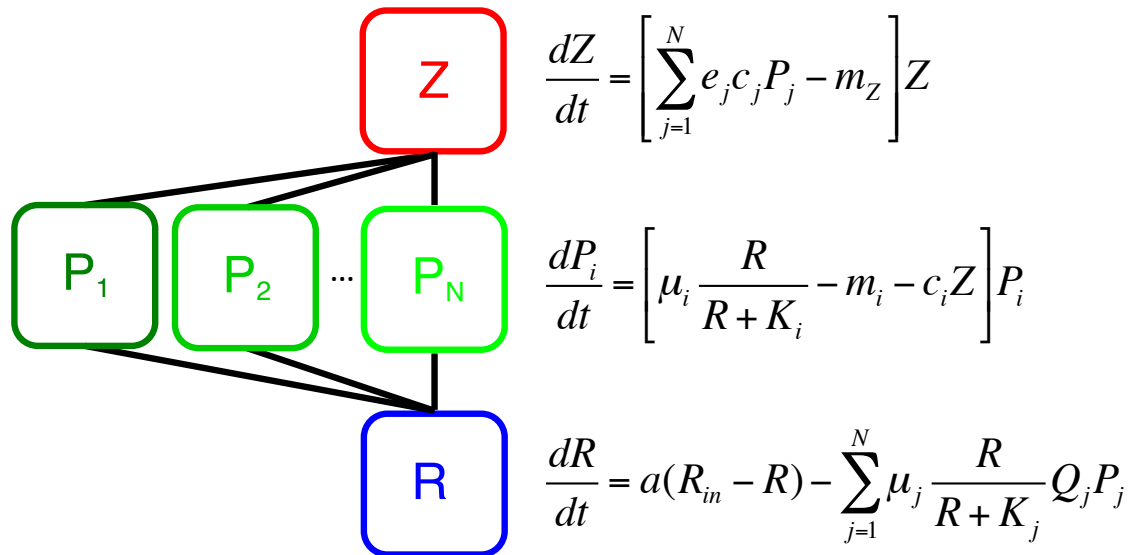


$$\frac{dN_i}{dt} = g(x_i; E(\vec{x}, \vec{N}))N_i$$

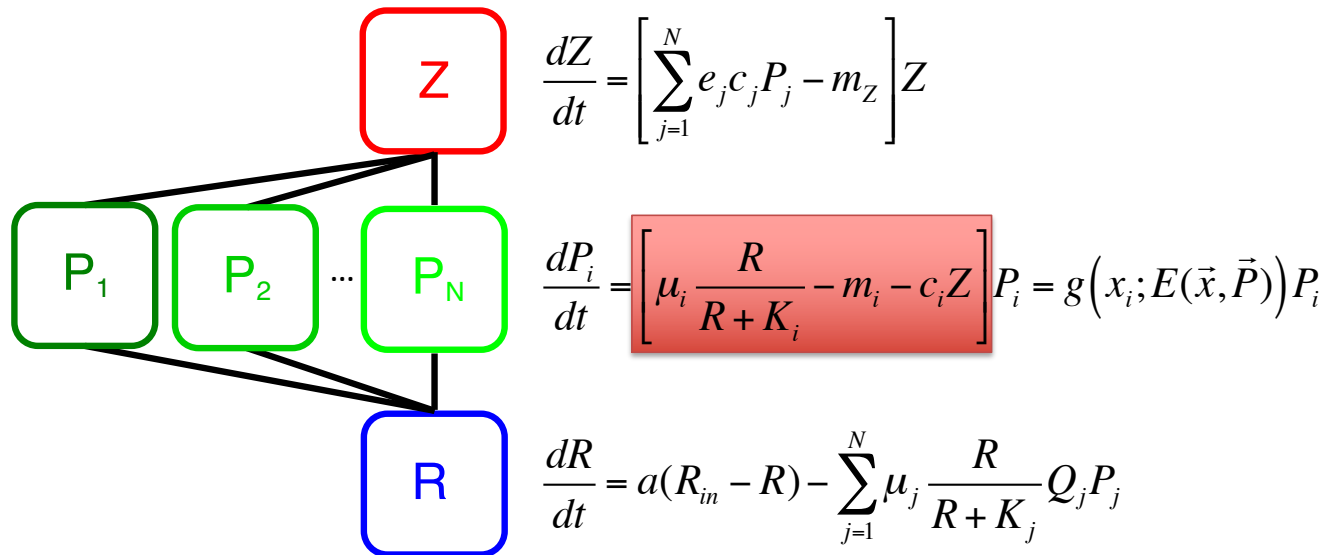
1) Start with a mechanistic model of growth



2) Generalize to \mathcal{N} strategies



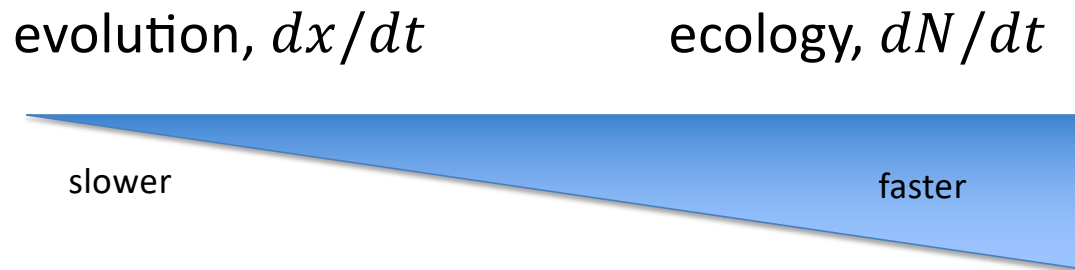
3) Identify fitness



How should we define fitness for general ecological scenarios? (Metz et al. 1992 *TREE*)

	Constant environment	Periodic environment	Aperiodic environment
Unstructured population	$\frac{dN}{dt} = gN$ fitness = g	$\frac{dN}{dt} = g(t)N$ fitness = $\frac{1}{\tau} \int_0^{\tau} g(t) dt$	$\frac{dN}{dt} = g(t)N$ fitness = $\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^{\tau} g(t) dt$
Extensively structured population	$\frac{d\vec{N}}{dt} = G\vec{N}$ fitness = $\max[\text{Re}[\lambda(G)]]$ (dominant eigenvalue) (Caswell 2001)	$\frac{d\vec{N}}{dt} = G(t)\vec{N}$ fitness = ... (dominant Floquet exponent) (Klausmeier 2008)	$\frac{d\vec{N}}{dt} = G(t)\vec{N}$ fitness = ... (dominant Lyapunov exponent) (Metz et al. 1992)

Separation of Time Scales in Adaptive Dynamics



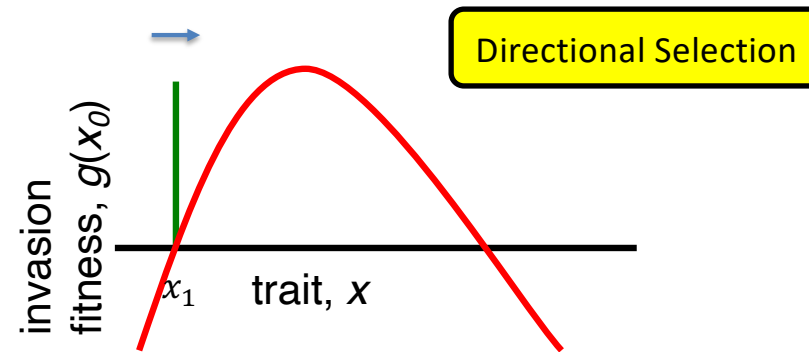
$$\frac{dN_i}{dt} = g(x_i)N_i = 0$$

Either: $N_i = 0$ or $g(x_i) = 0$

(Geritz *et al.* 1998, *Evol Ecol*)

Evolutionary Equilibria & Their Stability

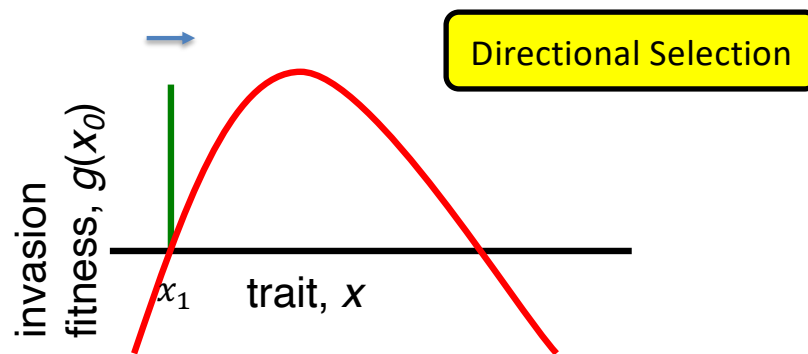
Eco Eq:
 $g(x_1) = 0$
 $\left. \frac{\partial g}{\partial x_0} \right|_{x_1} > 0$



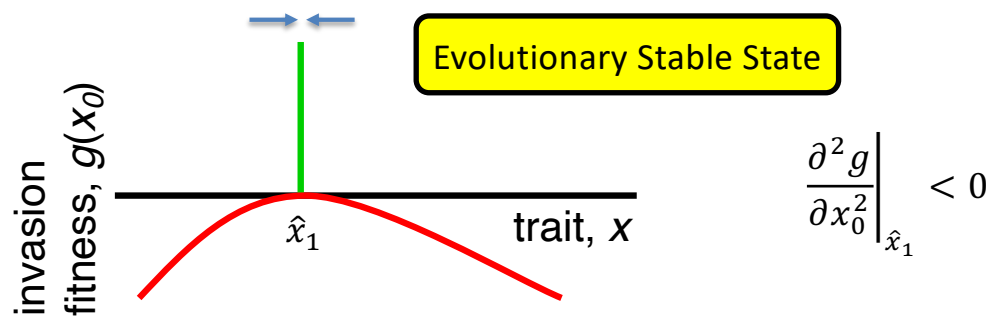
(Geritz *et al.* 1998, *Evol Ecol*)

Evolutionary Equilibria & Their Stability

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Eco Eq:
 $g(\hat{x}_1) = 0$
Evo Eq:
 $\left. \frac{\partial g}{\partial x_0} \right|_{\hat{x}_1} = 0$

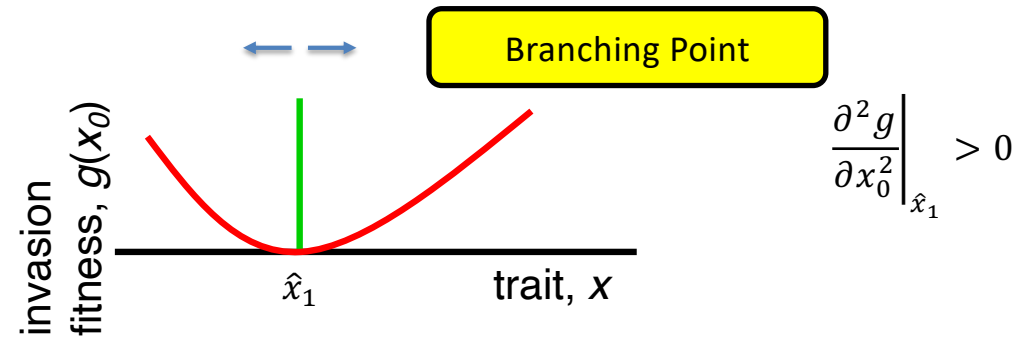


(Geritz *et al.* 1998, *Evol Ecol*)

Evolutionary Equilibria & Their Stability

Eco Eq:
 $g(\hat{x}_1) = 0$

Evo Eq:
 $\left. \frac{\partial g}{\partial x_0} \right|_{\hat{x}_1} = 0$

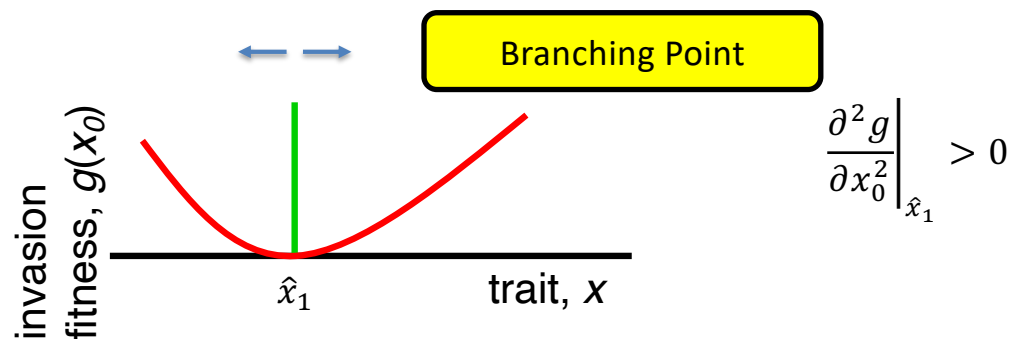


(Geritz *et al.* 1998, *Evol Ecol*)

Evolutionary Equilibria & Their Stability

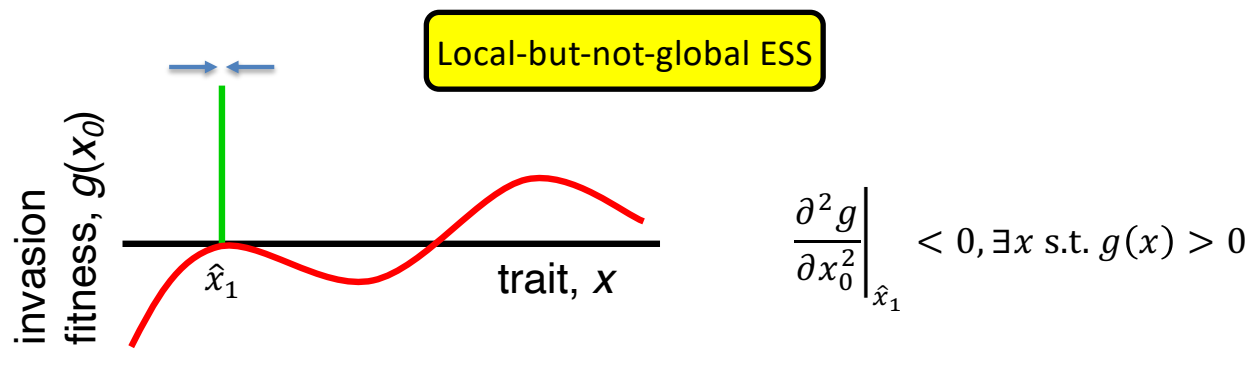
Eco Eq:
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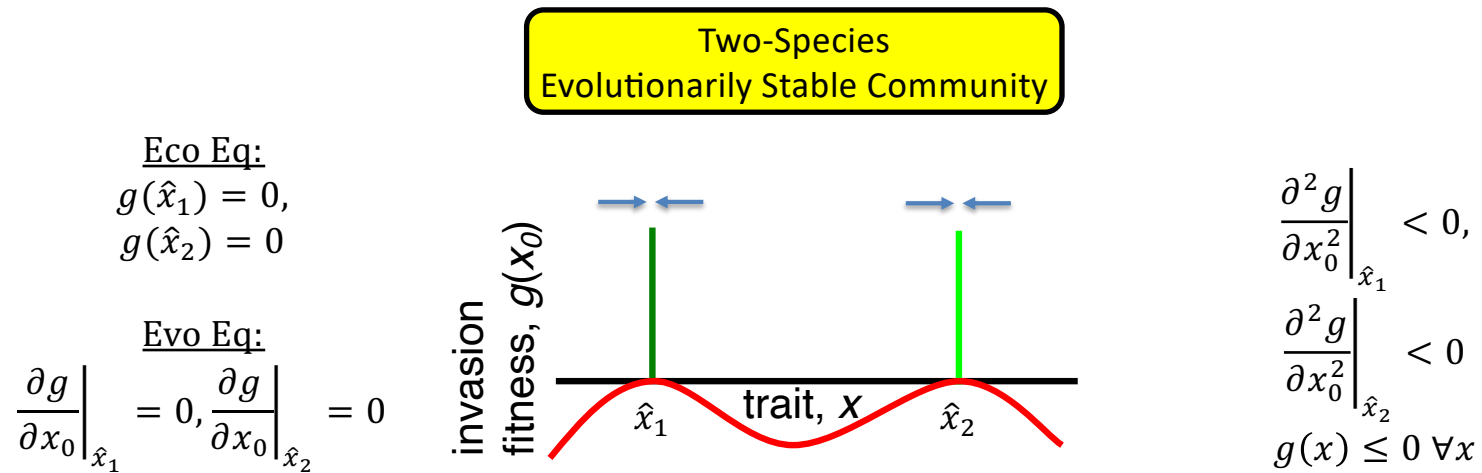
Eco Eq:
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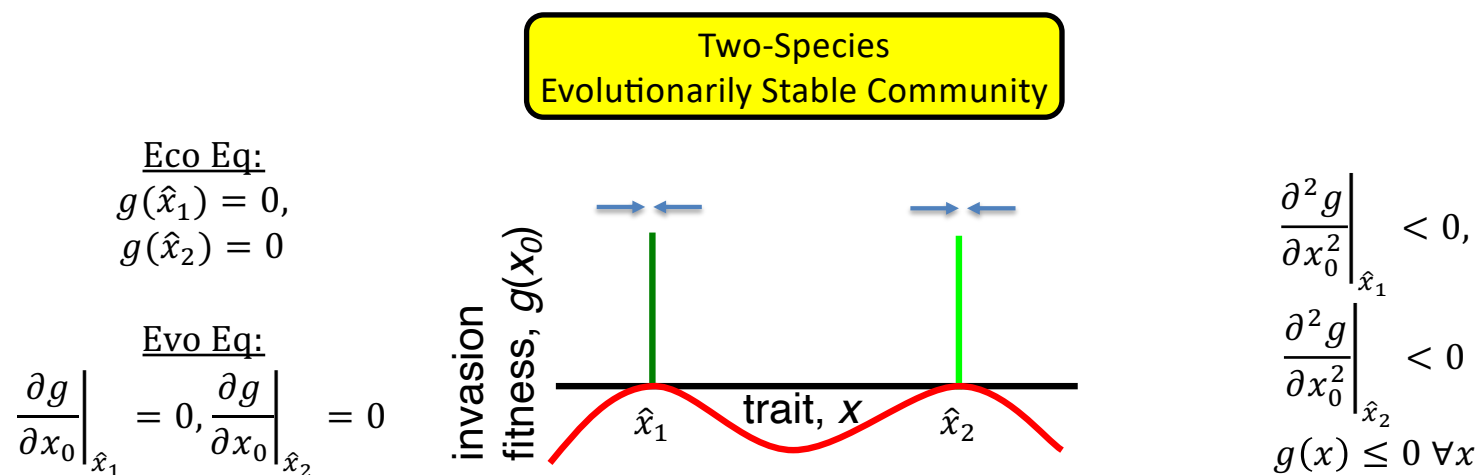
(Geritz *et al.* 1998, *Evol Ecol*)

Evolutionary Equilibria & Their Stability



(Geritz *et al.* 1998, *Evol Ecol*)

Evolutionary Equilibria & Their Stability

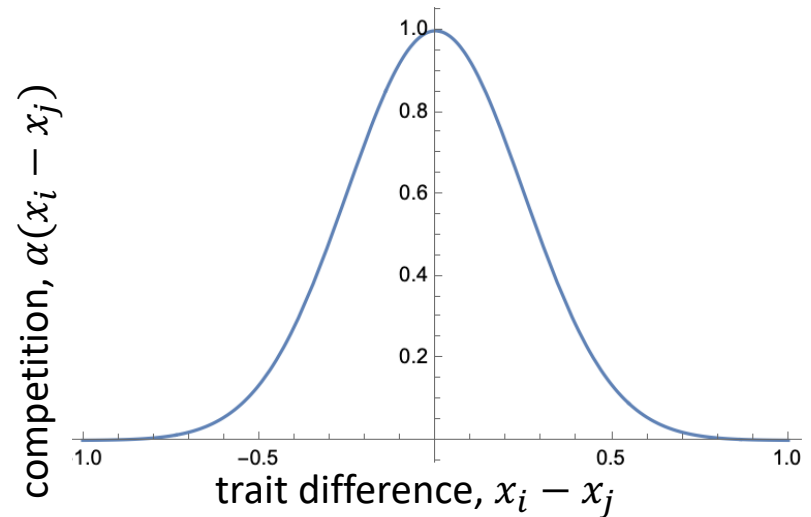
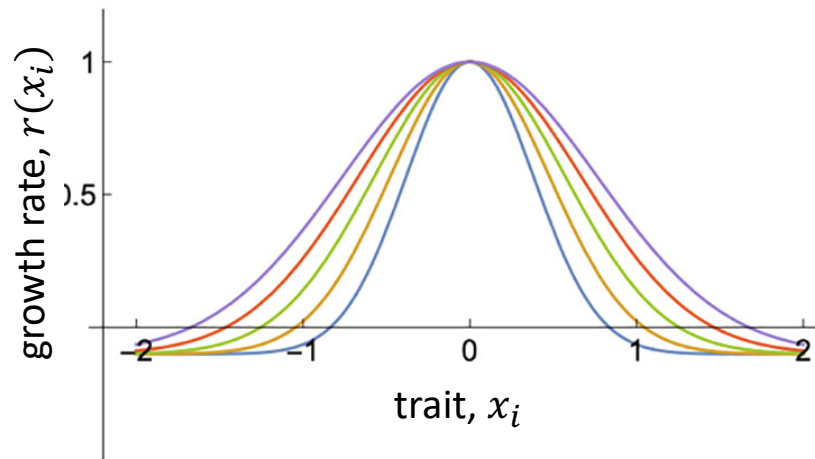


☞ An ESC is an endpoint of evolution AND community assembly
(Edwards *et al.* 2018 *Ecol Let*)

(Geritz *et al.* 1998, *Evol Ecol*)

Example: Lotka-Volterra Competition

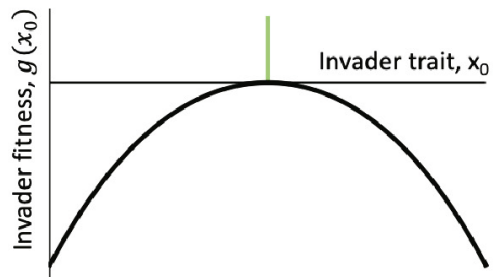
$$\frac{dN_i}{dt} = g(x_i)N_i = \left(r(x_i) - \sum_{j=1}^{\mathcal{N}} \alpha(x_i, x_j)N_j \right) N_i$$



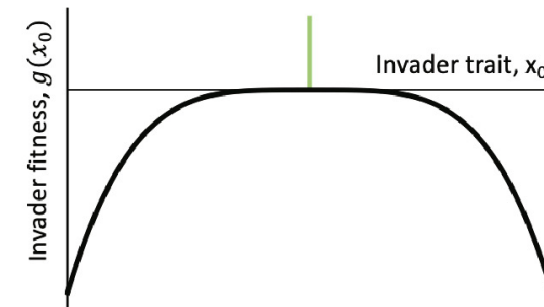
(Ranjan & Klausmeier 2022 *JTB*)

Expanding the resource distribution

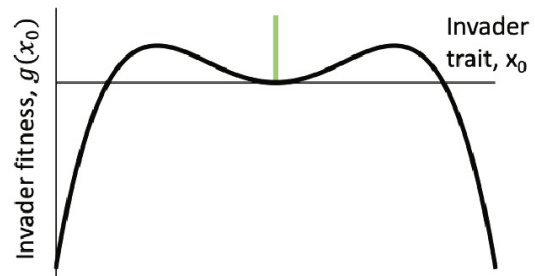
a) Before bifurcation point



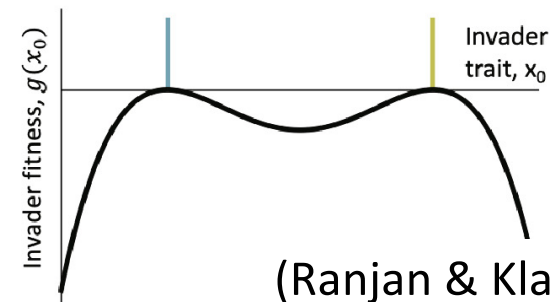
b) At the bifurcation point



c) Beyond bifurcation point



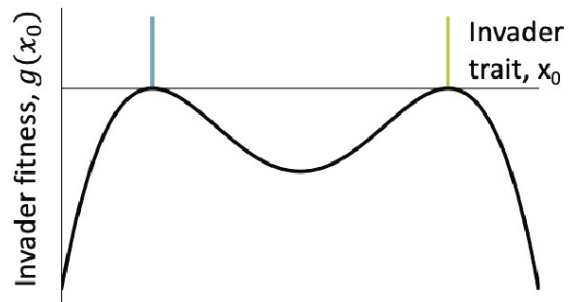
d) Two-species ESC



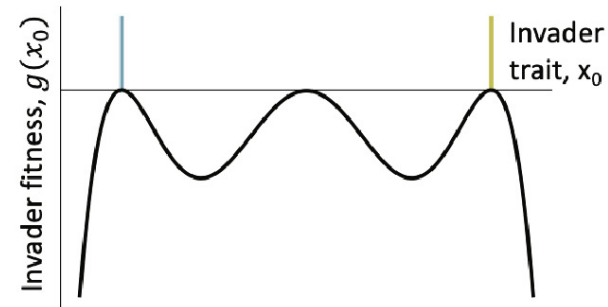
(Ranjan & Klausmeier 2022 *JTB*)

Expanding the resource distribution

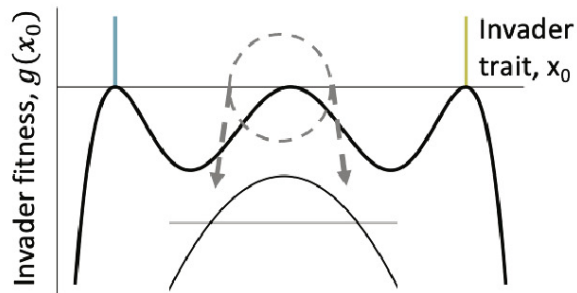
a) Before the bifurcation point



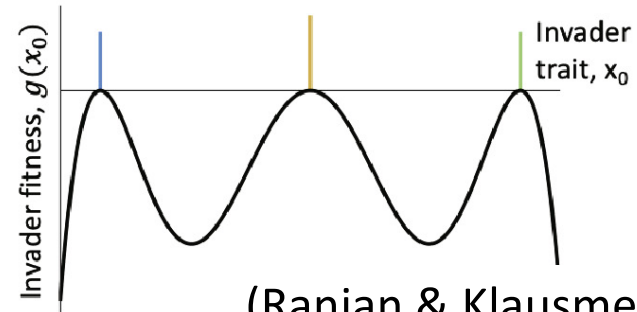
b) At the bifurcation point



c) Beyond the bifurcation point

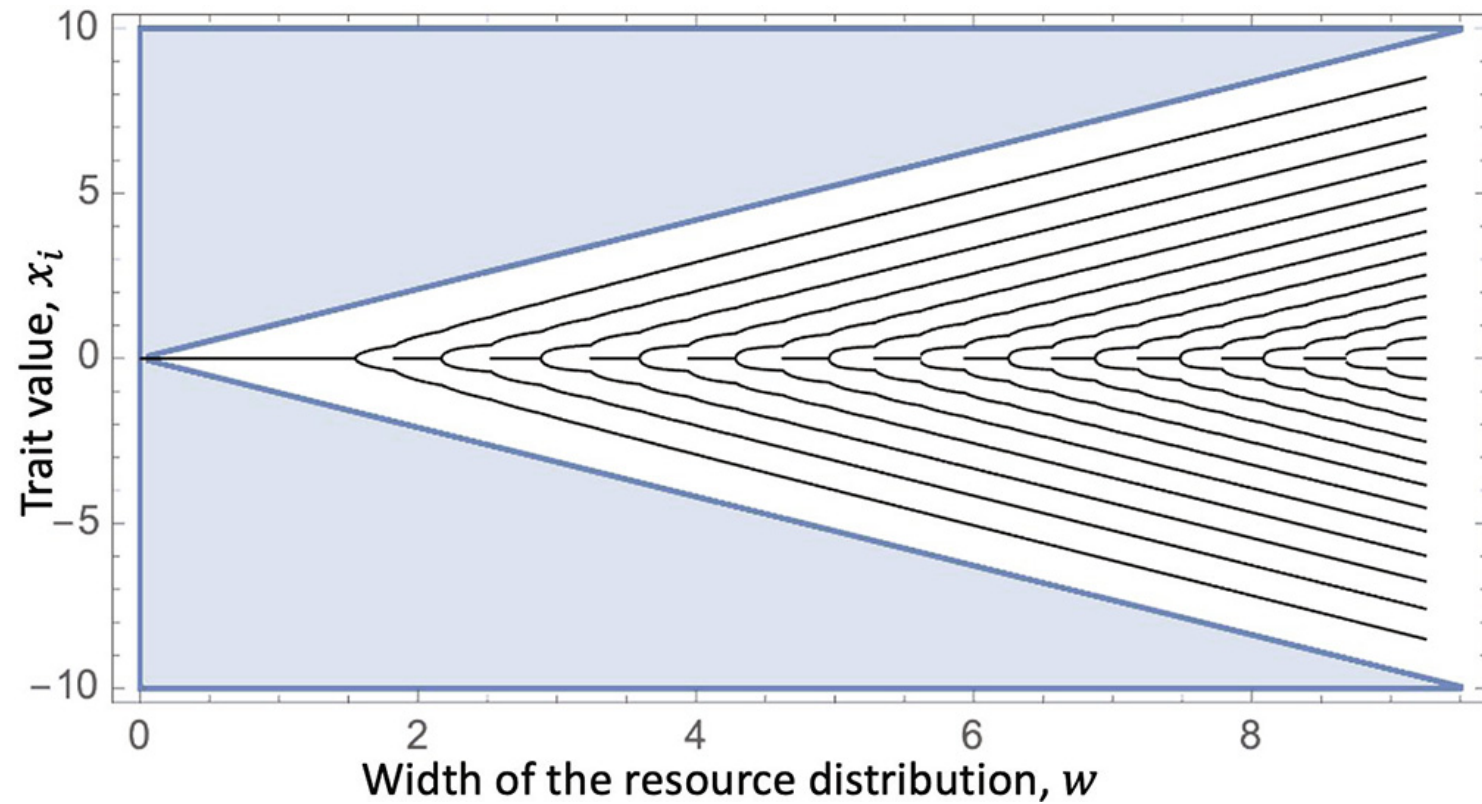


d) Three-species ESC



(Ranjan & Klausmeier 2022 *JTB*)

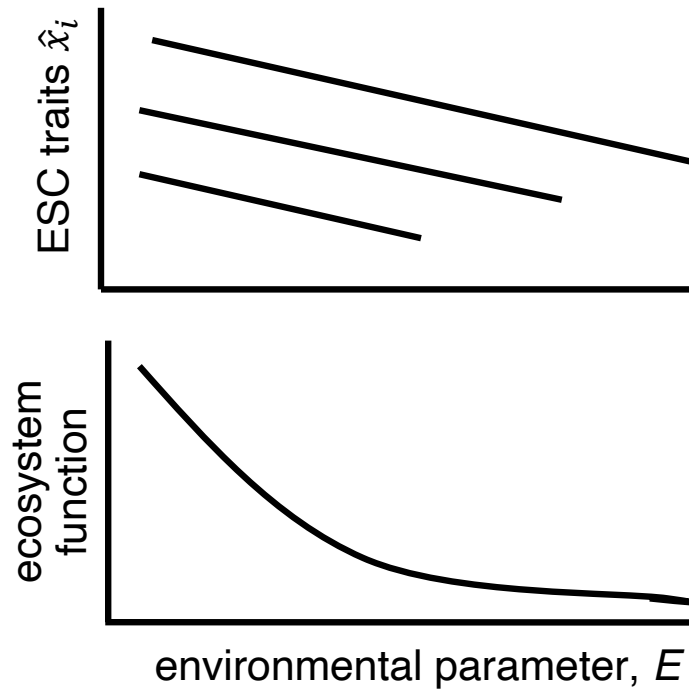
Example: Lotka-Volterra Competition



(Ranjan & Klausmeier 2022 *JTB*)

Eco-evolutionary Bifurcation Diagrams

- How do community structure (diversity, species traits) and ecosystem functions depend on abiotic environmental parameters?
- How will ecosystems reorganize in the face of human impacts?



Separation of Time Scales in Adaptive Dynamics

environmental change,
 dE/dt

evolution, dx/dt

ecology, dN/dt

slower

faster



Predicting Population Responses to Environmental Change



OPINION

Are We in the Midst Of a Sixth Mass Extinction?

A Tally of Life Under Threat

The International Union for Conservation of Nature has evaluated 52,205 species, depicted here, for their ability to survive. [Related Article »](#)

Each symbol represents 100 species assessed:

THREATENED

NOT THREATENED



Stark Indicators Of Extinction Risks

Because most **known species** of birds, mammals and amphibians have been evaluated, scientists are confident about the percentage of each group that is threatened.

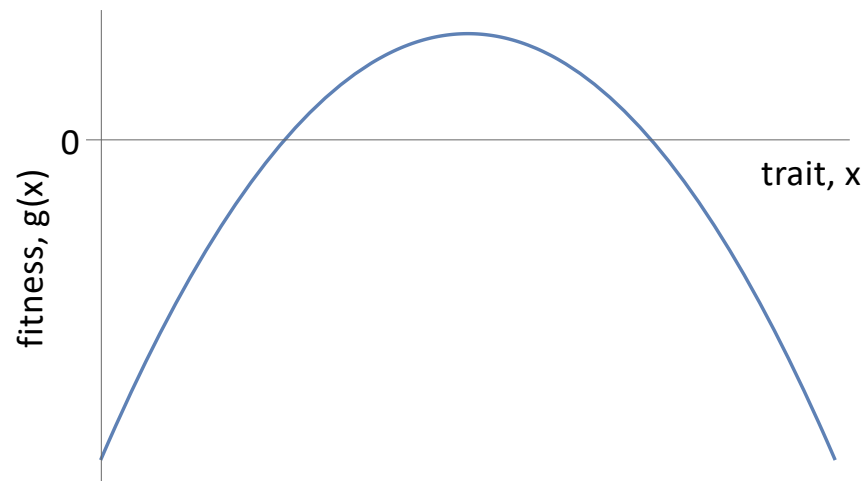
Bill Marsh, New York Times

III. Evolutionary Rescue



evolutionary rescue — the recovery and persistence of a population through natural selection acting on heritable variation

Evolution in a constant environment



Quantitative Genetics:

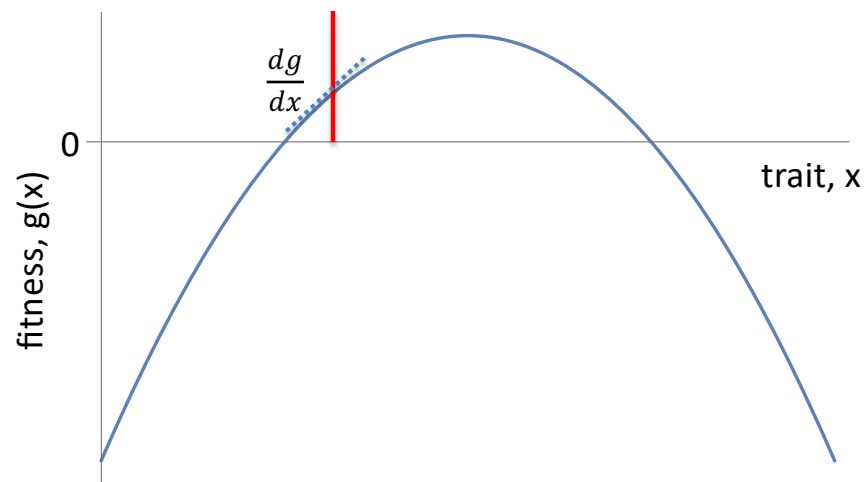
$$\frac{dx}{dt} = V \frac{dg}{dx}$$

Population Dynamics:

$$\frac{dN}{dt} = g(x)N$$

Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Kareiva P, Kingsolver J, Huey R (eds) Biotic Interactions and Global Change. Sinauer, pp 234–250

Evolution in a constant environment



Quantitative Genetics:

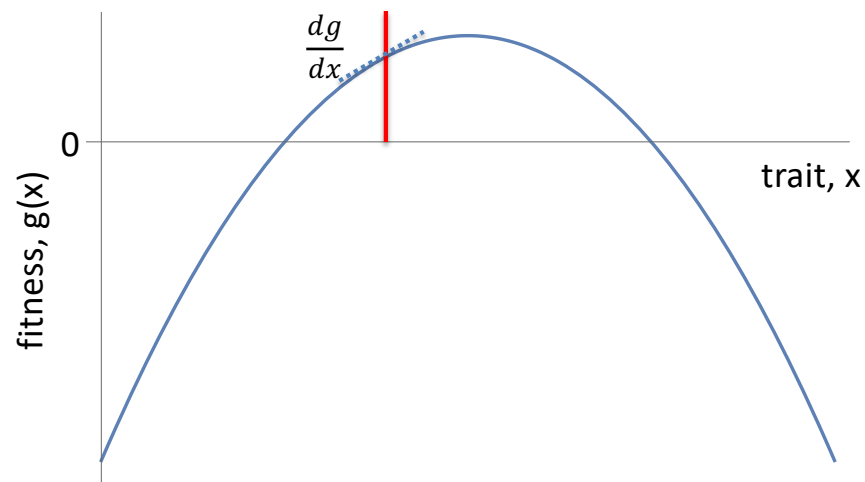
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Evolution in a constant environment



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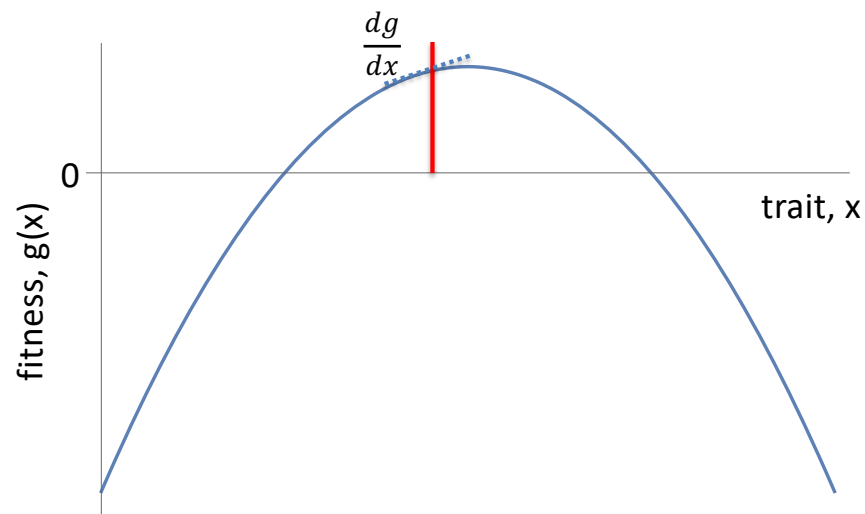
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Evolution in a constant environment



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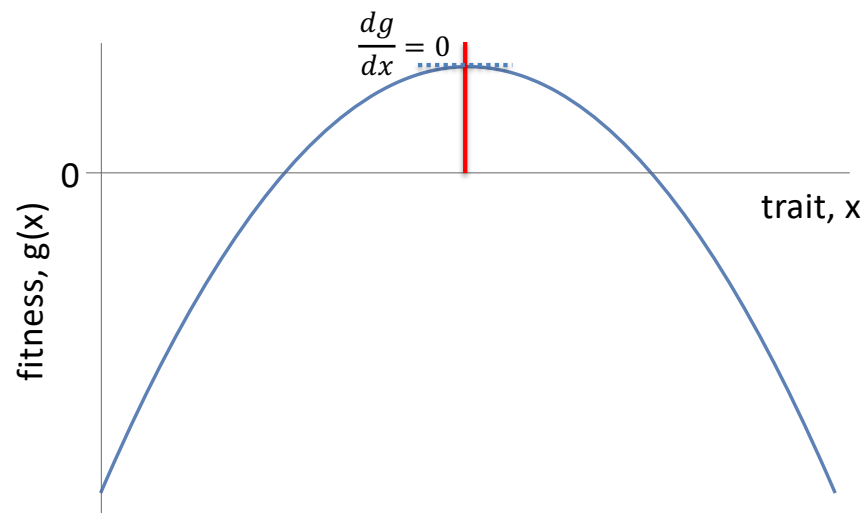
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Evolution in a constant environment



Quantitative Genetics:

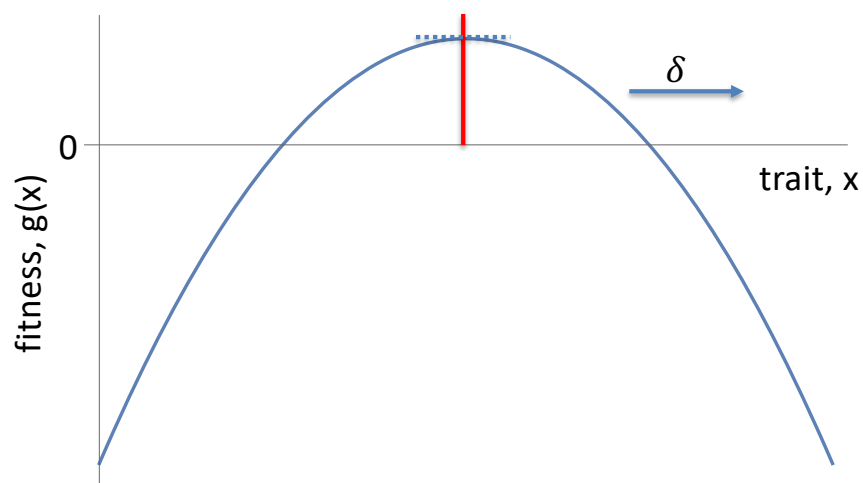
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Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Kareiva P, Kingsolver J, Huey R (eds) Biotic Interactions and Global Change. Sinauer, pp 234–250

Evolution in a changing environment



Quantitative Genetics:

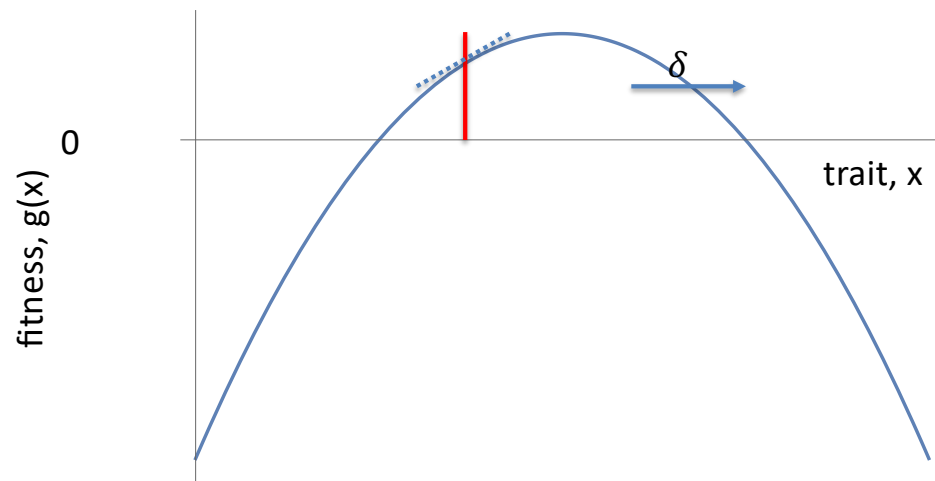
$$\frac{dx}{dt} = V \frac{dg}{dx}$$

Population Dynamics:

$$\frac{dN}{dt} = g(x)N$$

Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Kareiva P, Kingsolver J, Huey R (eds) Biotic Interactions and Global Change. Sinauer, pp 234–250

Evolution in a changing environment



Quantitative Genetics:

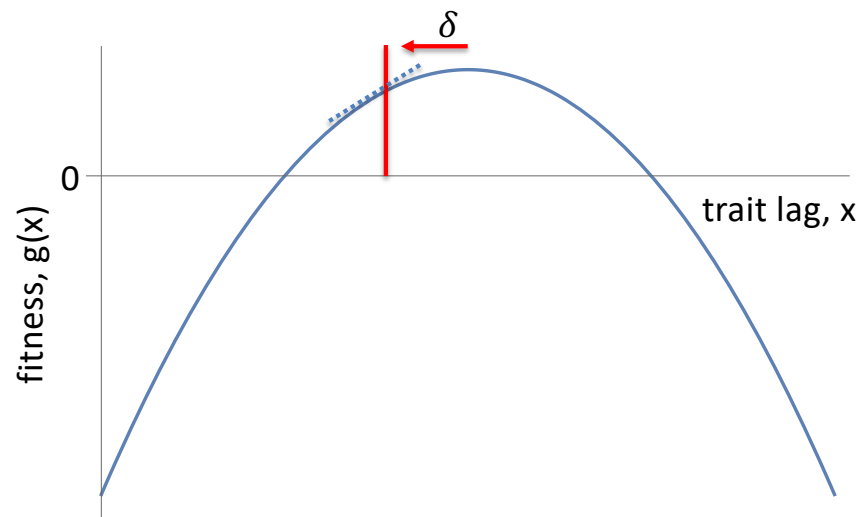
$$\frac{dx}{dt} = V \frac{dg}{dx}$$

Population Dynamics:

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Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Kareiva P, Kingsolver J, Huey R (eds) Biotic Interactions and Global Change. Sinauer, pp 234–250

Clever trick: moving frame of reference



Quantitative Genetics:

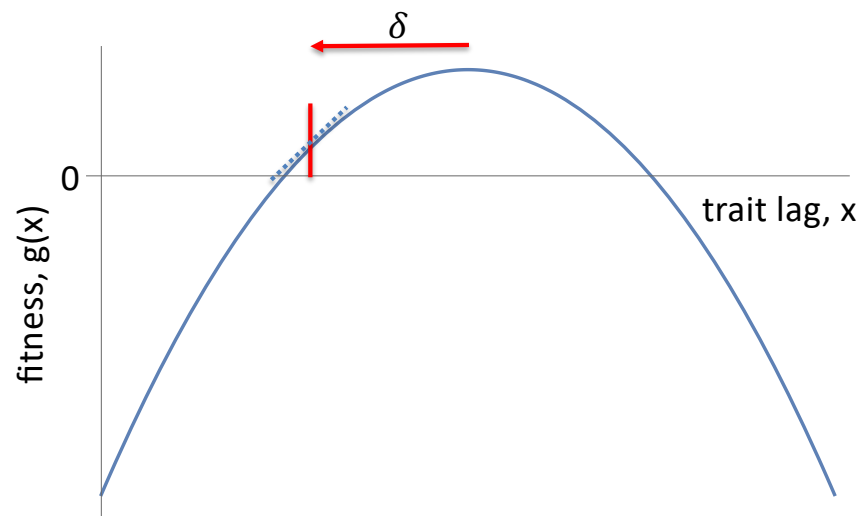
$$\frac{dx}{dt} = V \frac{dg}{dx} - \delta$$

Population Dynamics:

$$\frac{dN}{dt} = g(x)N$$

Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Kareiva P, Kingsolver J, Huey R (eds) Biotic Interactions and Global Change. Sinauer, pp 234–250

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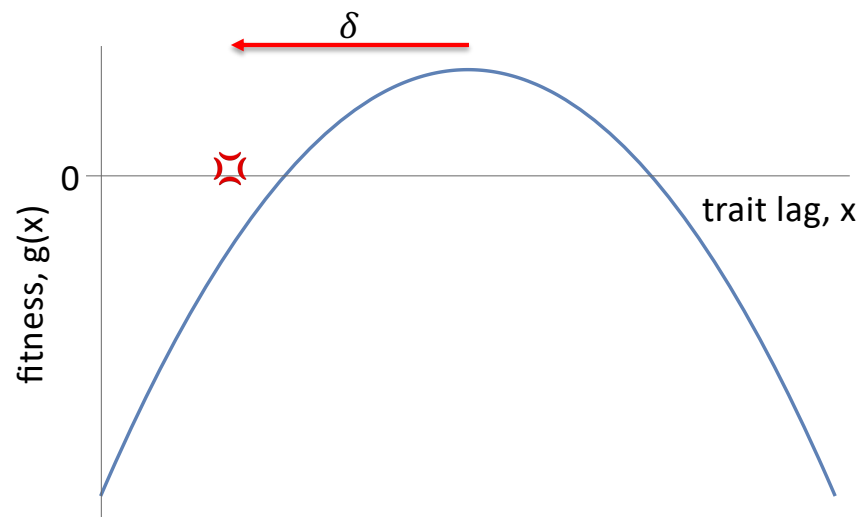
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Quantitative Genetics:

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Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Kareiva P, Kingsolver J, Huey R (eds) Biotic Interactions and Global Change. Sinauer, pp 234–250

Lynch & Lande (1993) Conclusions

- 1) Environmental change causes trait to lag optimum
- 2) Equilibrium lag increases linearly with rate of environmental change δ
- 3) Increased genetic variance V helps species keep up
- 4) There is a critical rate of environmental change δ_c where $g(\hat{x}) = 0$ that leads to extinction

Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Kareiva P, Kingsolver J, Huey R (eds) Biotic Interactions and Global Change. Sinauer, pp 234–250

Research



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Accepted: 24 March 2020

One contribution of 17 to a theme issue 'Integrative research perspectives on marine conservation'.

Subject Areas:
ecology, evolution

Keywords:
climate change, eco-evolutionary dynamics, environmental change, evolutionary rescue, moving optimum, quantitative genetics


Ecological limits to evolutionary rescue

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¹W. K. Kellogg Biological Station, Michigan State University, 3700 East Gull Lake Drive, Hickory Corners, MI 49060, USA

²Department of Plant Biology, ³Department of Integrative Biology, and ⁴Program in Ecology, Evolution and Behavior, Michigan State University, East Lansing, MI, USA

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 CAK, 0000-0002-6987-5871; EL, 0000-0001-7736-6332

Environments change, for both natural and anthropogenic reasons, which can threaten species persistence. Evolutionary adaptation is a potentially powerful mechanism to allow species to persist in these changing environments. To determine the conditions under which adaptation will prevent extinction (evolutionary rescue), classic quantitative genetics models have assumed a constantly changing environment. They predict that species traits will track a moving environmental optimum with a lag that approaches a constant. If fitness is negative at this lag, the species will go extinct. There have been many elaborations of these models incorporating increased genetic realism. Here, we review and explore the consequences of four ecological complications: non-quadratic fitness functions, interacting density- and trait-dependence, species interactions and fundamental limits to adaptation. We show that non-quadratic fitness functions can result in evolutionary tipping points and existential crises, as can the interaction between density- and trait-dependent mortality. We then review the literature on how interspecific interactions affect adaptation and persistence. Finally, we suggest an alternative theoretical framework that considers bounded environmental change and fundamental limits to adaptation. A research programme that combines theory and experiments and integrates across organizational scales will be needed to predict whether adaptation will prevent species extinction in changing environments.

This article is part of the theme issue 'Integrative research perspectives on marine conservation'.

Ecological Complications

- 1) Non-quadratic fitness functions
- 2) Population regulation
- 3) Community context
- 4) Fundamental niche limits

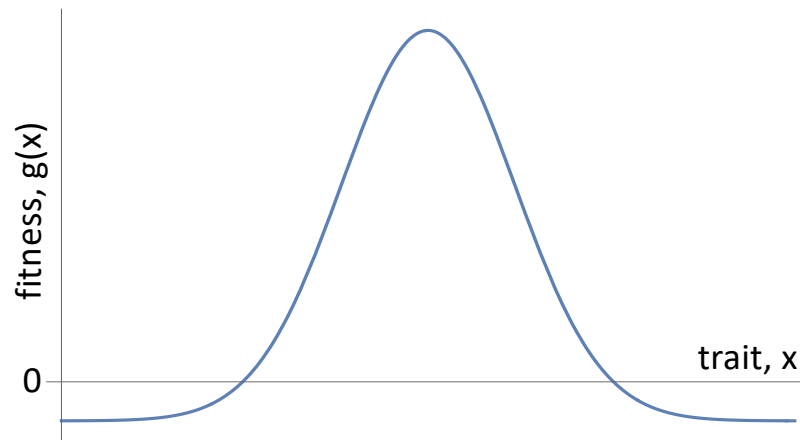
Ecological Complications

- 1) Non-quadratic fitness functions
- 2) Population regulation
- 3) Community context
- 4) Fundamental niche limits

Gaussian Fitness Function



Matt Osmond
(Toronto)



Quantitative Genetics:

$$\frac{dx}{dt} = V \frac{dg}{dx} - \delta$$

Population Dynamics:

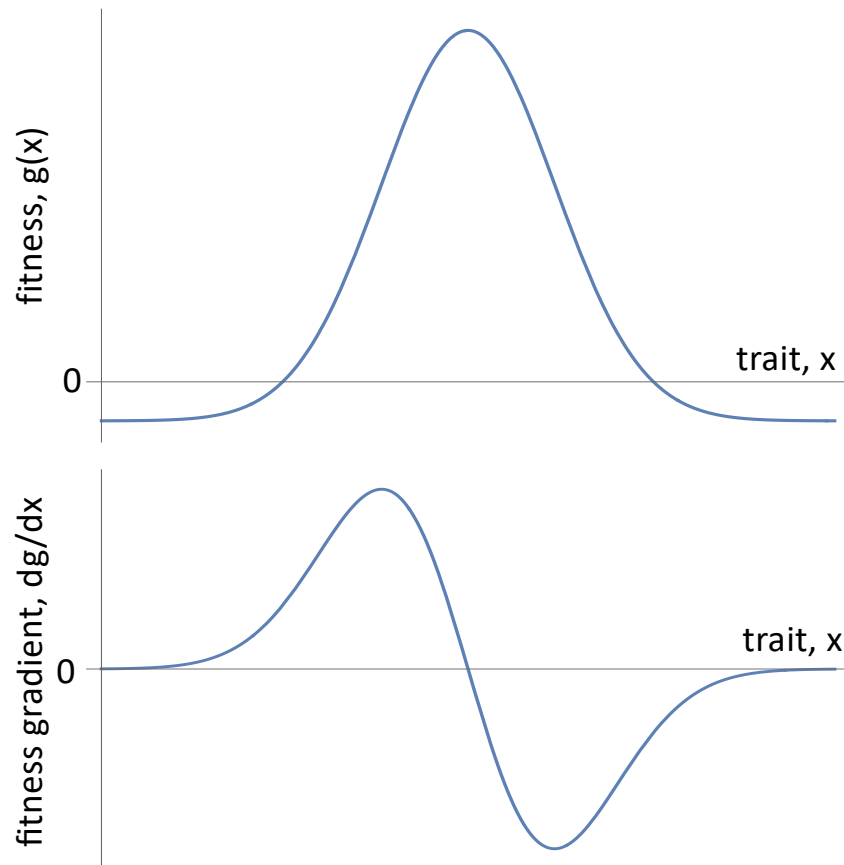
$$\frac{dN}{dt} = g(x)N$$

Osmond MM, Klausmeier CA (2017) An evolutionary tipping point in a changing environment. *Evolution* 71:2930–2941

Gaussian Fitness Function



Matt Osmond
(Toronto)



Quantitative Genetics:

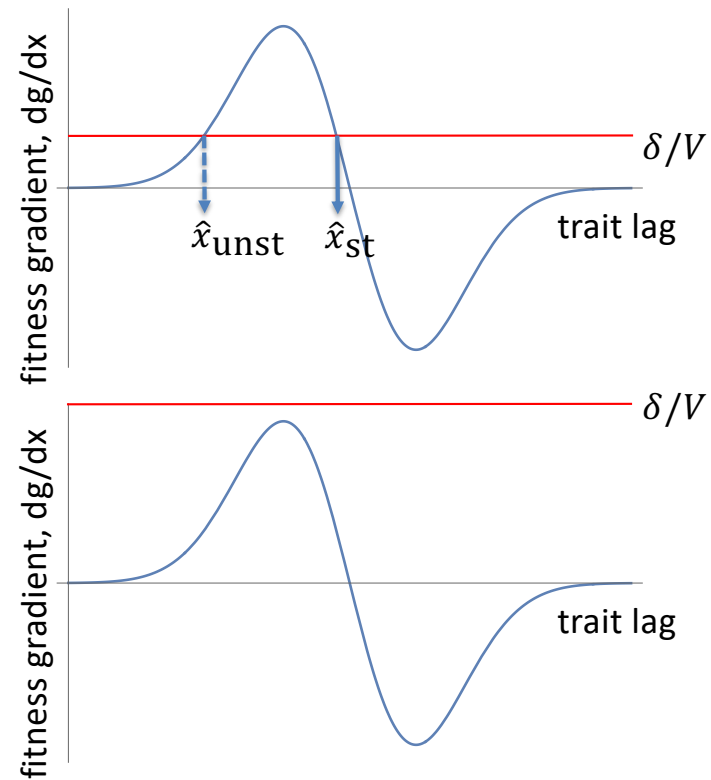
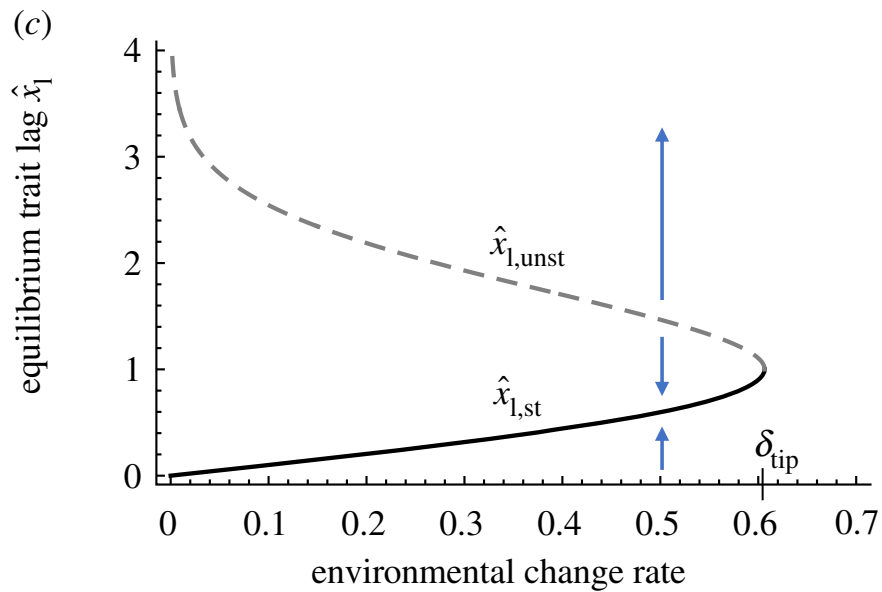
$$\frac{dx}{dt} = V \frac{dg}{dx} - \delta$$

Population Dynamics:

$$\frac{dN}{dt} = g(x)N$$

Osmond MM, Klausmeier CA (2017) An evolutionary tipping point in a changing environment. *Evolution* 71:2930–2941

Existential Crises & Evolutionary Tipping Points



Osmond MM, Klausmeier CA (2017) An evolutionary tipping point in a changing environment. *Evolution* 71:2930–2941

Ecological Complications

- 1) Non-quadratic fitness functions
- 2) Population regulation
- 3) Community context
- 4) Fundamental niche limits

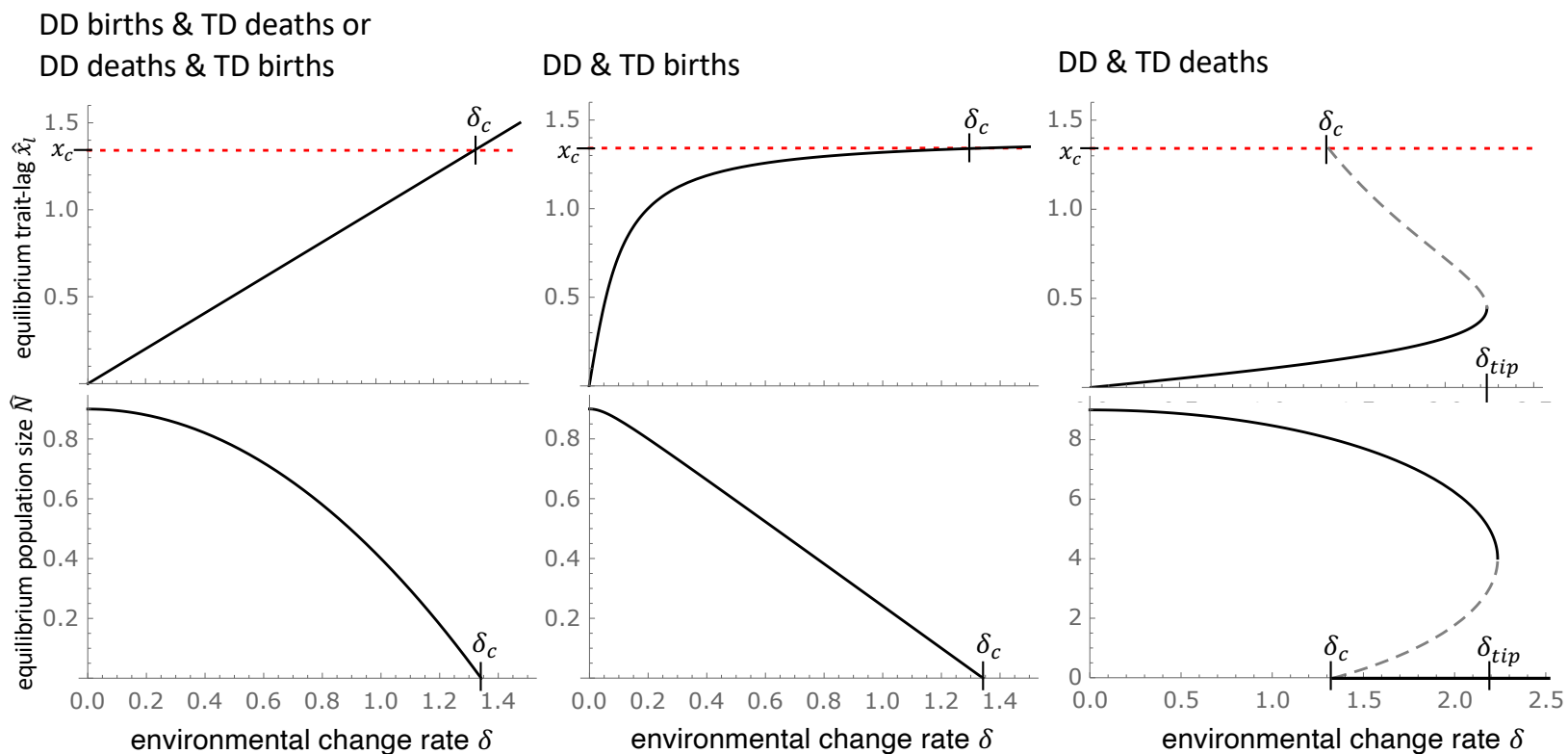
Population Regulation

Density-dependence (DD) & trait-dependence (TD) of growth can each affect births & deaths

Trait-Dependence	Density-Dependence	Growth Rate, $g(x, E, N)$
Births	Deaths	$\left(b_{\max} - \frac{(x - E)^2}{2\sigma_r^2} \right) - d(1 + N)$
Deaths	Births	$b(1 - N) - \left(d_{\min} + \frac{(x - E)^2}{2\sigma_r^2} \right)$
Births	Births	$\left(b_{\max} - \frac{(x - E)^2}{2\sigma_r^2} \right) (1 - N) - d$
Deaths	Deaths	$b - \left(d_{\min} + \frac{(x - E)^2}{2\sigma_r^2} \right) (1 + N)$

Klausmeier CA, Osmond MM, Kremer CT, Litchman E (2020) Ecological limits to evolutionary rescue. *Phil Trans R Soc B* 375:20190453

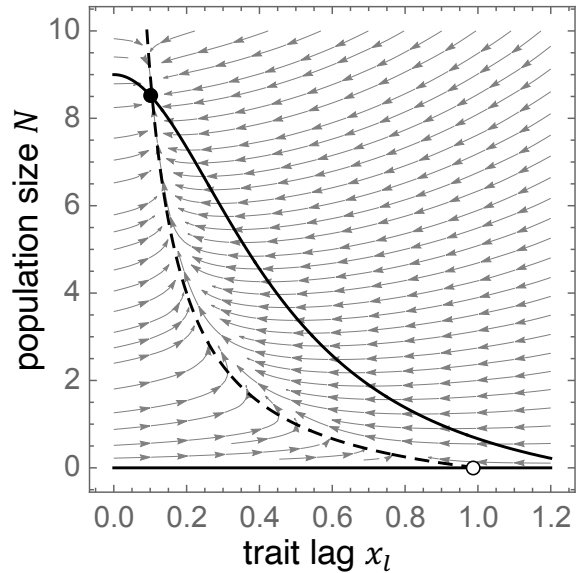
Equilibrium trait-lag & abundance



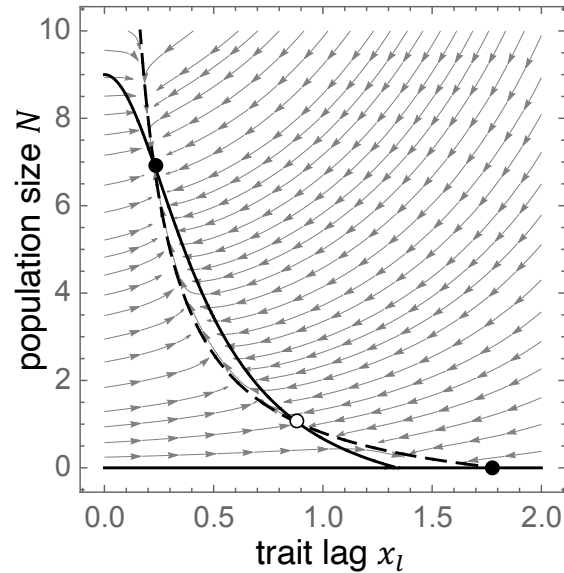
Klausmeier CA, Osmond MM, Kremer CT, Litchman E (2020) Ecological limits to evolutionary rescue. *Phil Trans R Soc B* 375:20190453

Eco-evo phase planes (DD & TD deaths)

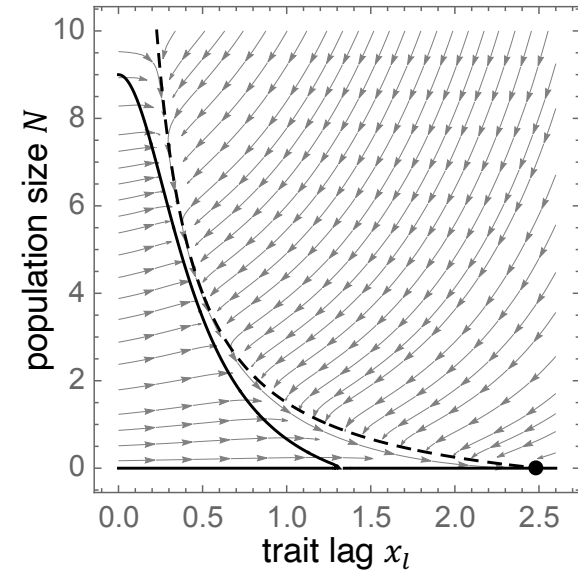
A) $\delta = 1$



B) $\delta = 1.8$



C) $\delta = 2.5$



Klausmeier CA, Osmond MM, Kremer CT, Litchman E (2020) Ecological limits to evolutionary rescue. *Phil Trans R Soc B* 375:20190453

Ecological Complications

- 1) Non-quadratic fitness functions
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Predator-prey

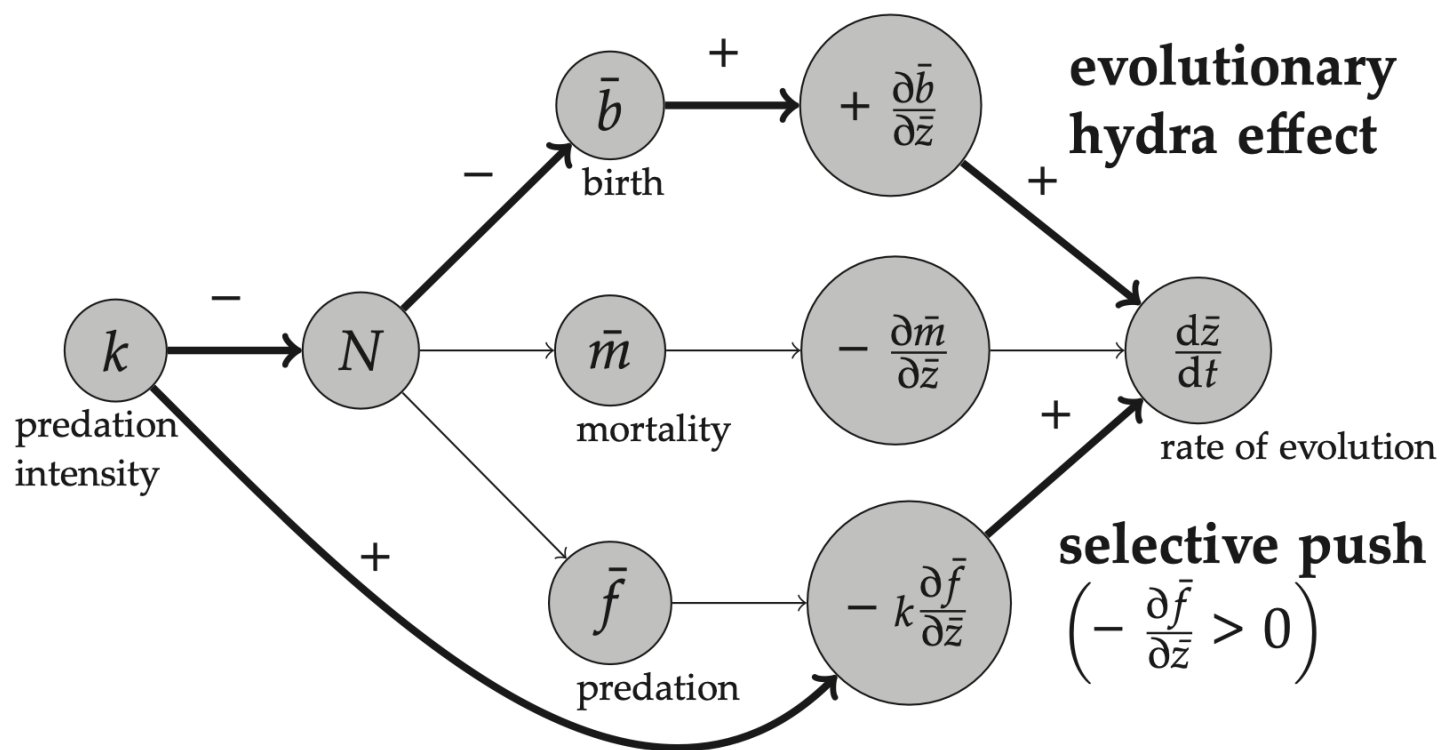
- Death due to predation from a generalist predator (k is strength of predation)

$$\frac{dN}{dt} = [\bar{b}(\bar{z}, N) - \bar{m}(\bar{z}, N) - k\bar{f}(\bar{z}, N)]N$$

- Does predator help prey *adapt* and *persist* in changing environments?

Osmond MM, Otto SP, Klausmeier CA (2017) When predators help prey adapt and persist in a changing environment. *American Naturalist* 190: 83–98.

Two ways predators help prey adapt



Osmond MM, Otto SP, Klausmeier CA (2017) When predators help prey adapt and persist in a changing environment. *American Naturalist* 190: 83–98.

Two ways predators help prey persist

Predators help prey persist if...

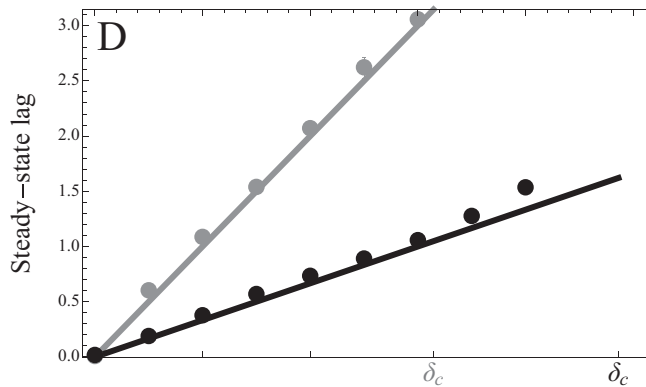
$$\frac{d\delta_c}{dk} = V_A \left(\frac{\partial^2 g}{\partial \bar{z} \partial k} + \frac{\partial \bar{z}_c}{\partial k} \cdot \frac{\partial^2 g}{\partial \bar{z}^2} \right) > 0$$

$$\left(\begin{array}{c} \text{selective} \\ \text{push} \end{array} \right) + \begin{array}{c} \text{evolutionary} \\ \text{hydra} \\ \text{effect} \end{array} \cdot \begin{array}{c} \text{fitness} \\ \text{function} \\ \text{curvature} \end{array} \right) > 0$$

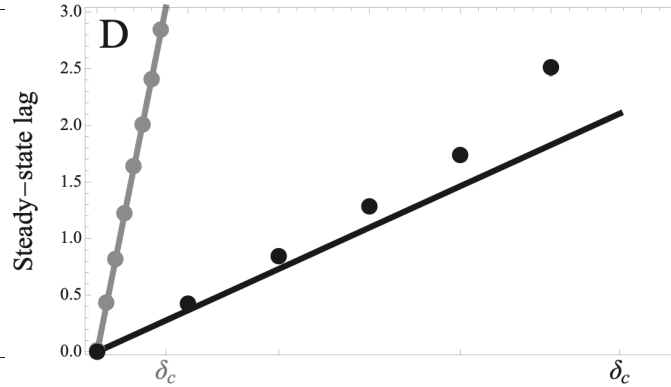
Osmond MM, Otto SP, Klausmeier CA (2017) When predators help prey adapt and persist in a changing environment. *American Naturalist* 190: 83–98.

Two ways predators help prey persist

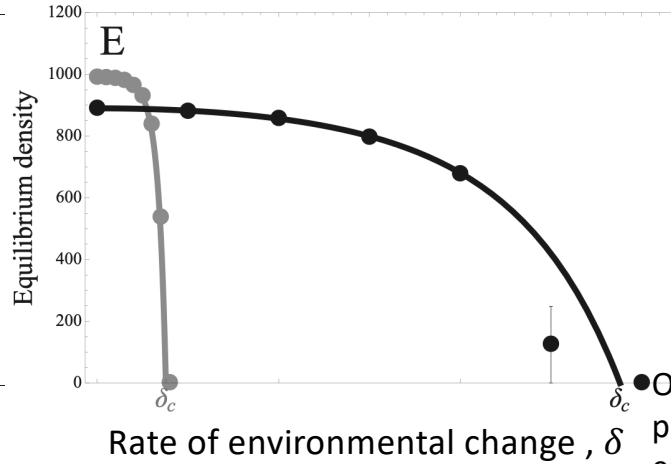
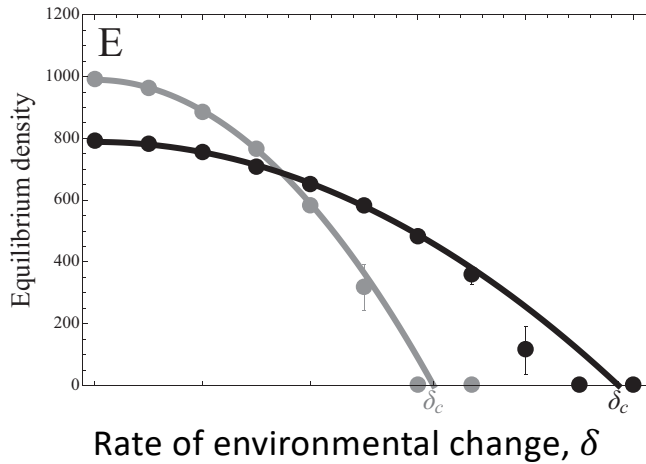
Selective push



Evol. hydra effect * positive curvature



— With predation
— Without predation

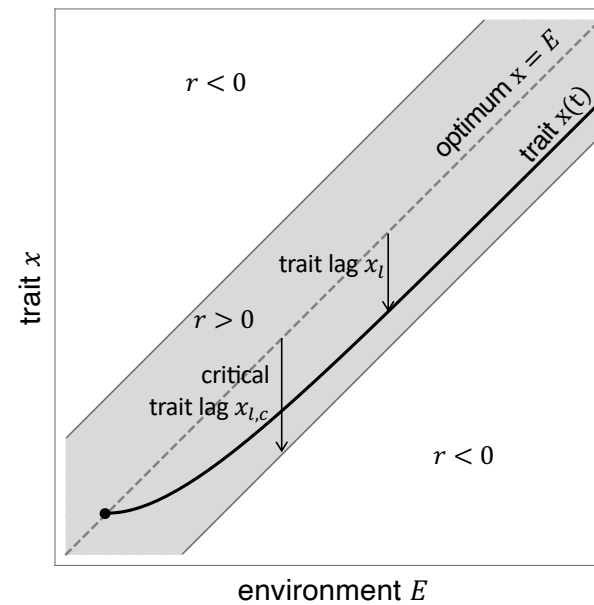
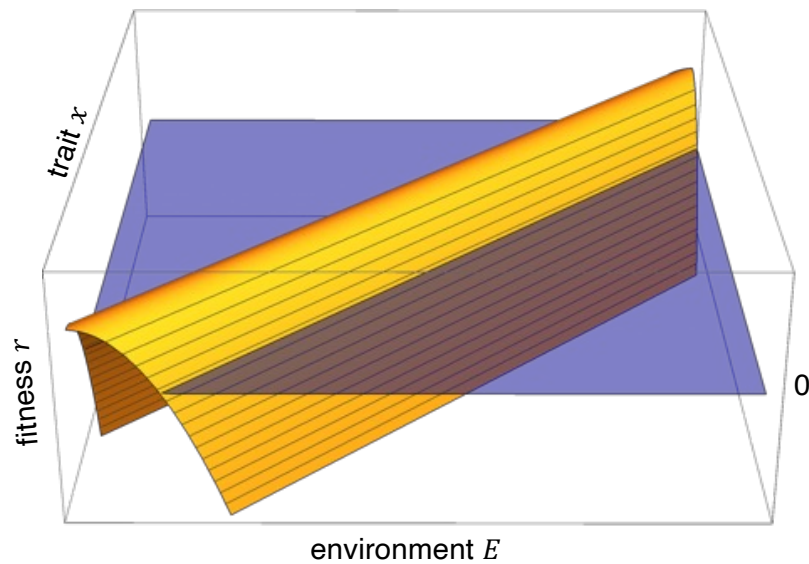


● Osmond MM, Otto SP, Klausmeier CA (2017) When predators help prey adapt and persist in a changing environment. *American Naturalist* 190: 83–98.

Ecological Complications

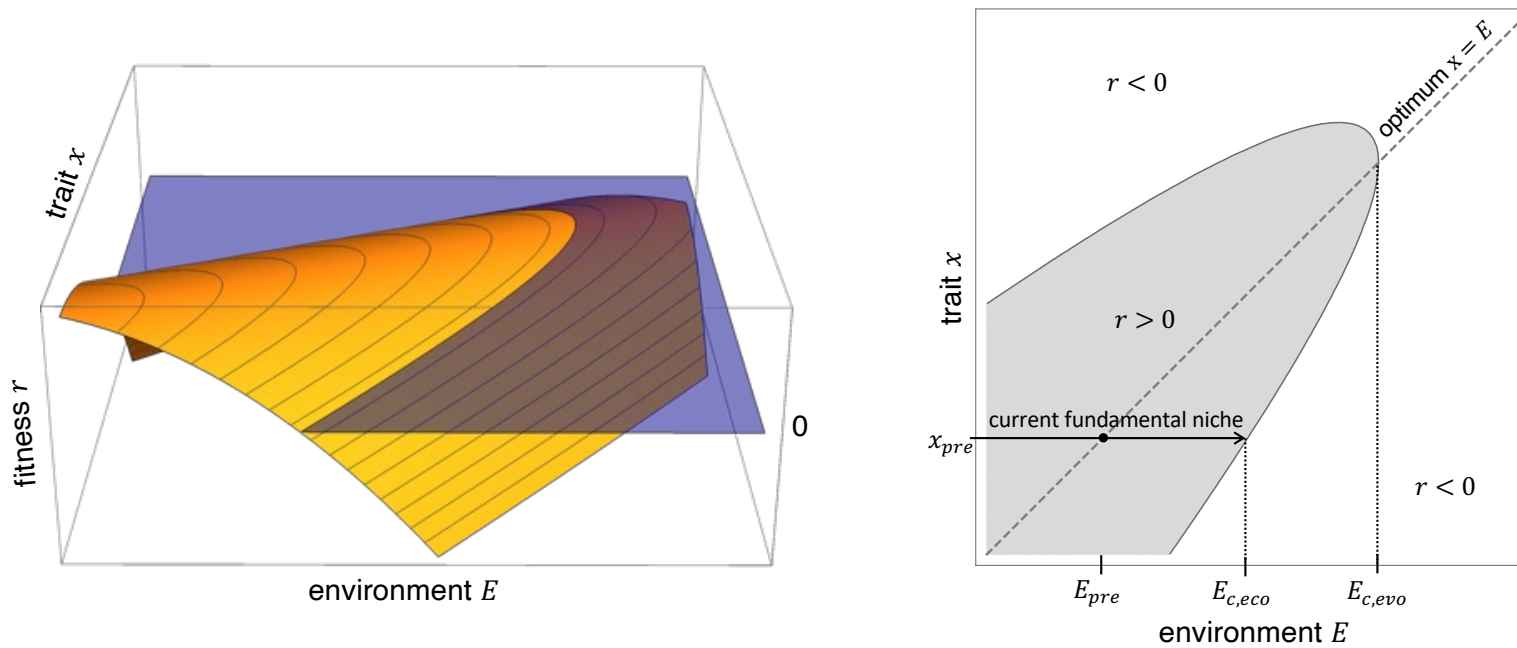
- 1) Non-quadratic fitness functions
- 2) Population regulation
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- 4) Fundamental niche limits

Continuous environmental change



Klausmeier CA, Osmond MM, Kremer CT, Litchman E (2020) Ecological limits to evolutionary rescue. *Phil Trans R Soc B* 375:20190453

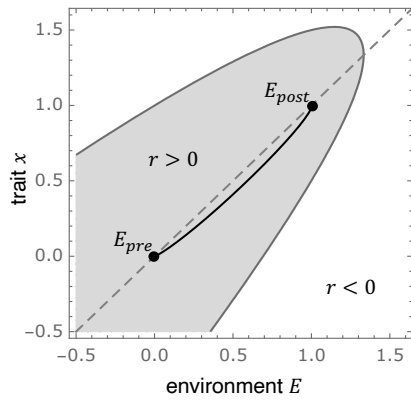
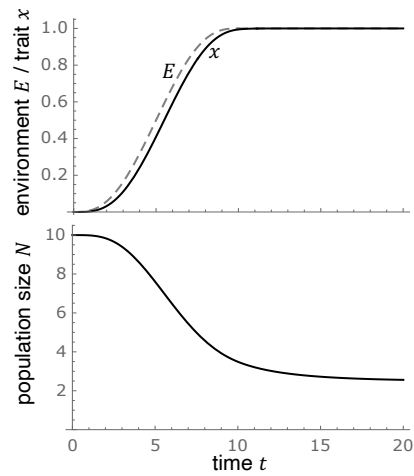
Fundamental niche limits



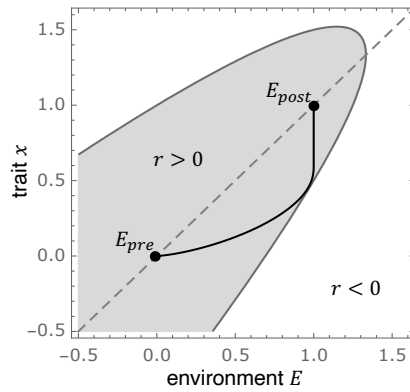
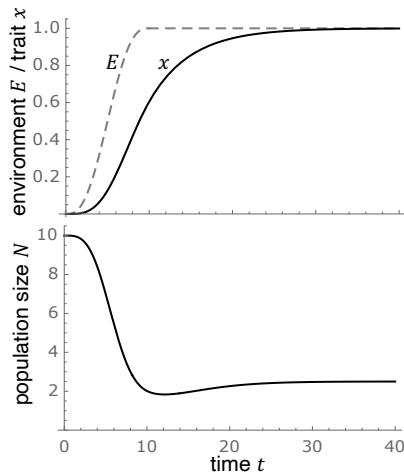
Klausmeier CA, Osmond MM, Kremer CT, Litchman E (2020) Ecological limits to evolutionary rescue. *Phil Trans R Soc B* 375:20190453

Speed of adaptation still matters

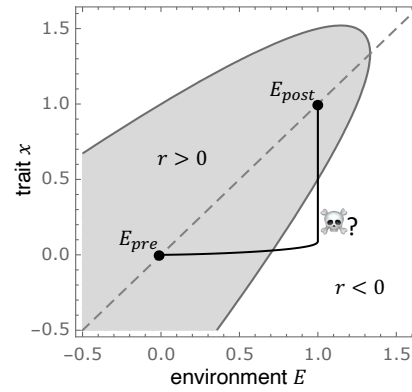
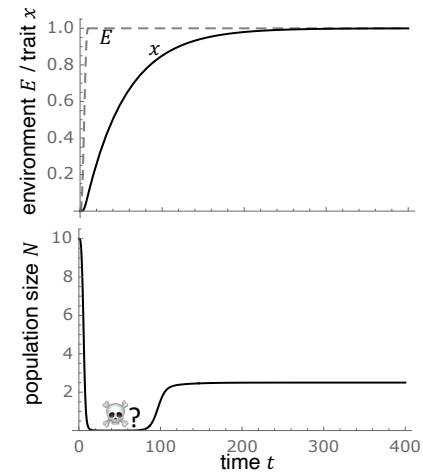
A) $V_a = 1$



B) $V_a = 0.1$




C) $V_a = 0.01$



III. Conclusions

- Non-quadratic fitness functions and the interplay between density-dependence & trait-dependence can lead to *evolutionary tipping points* and *existential crises*
- Community context matters: predators can help prey adapt & persist, trailing-edge competitors at heightened risk
 - Fundamental niche limits might constrain evolutionary rescue more than rate-dependent processes



ECOLOGICAL
LIMITS
TO

EVOLUTIONARY RESCUE

Norberg J, Urban MC, Vellend M, Klausmeier CA, Loeuille N (2012) Eco-evolutionary responses of biodiversity to climate change. *Nature Climate Change* 2: 747–751

Osmond MM, Klausmeier CA (2017) An evolutionary tipping point in a changing environment. *Evolution* 71: 2930–2941

Osmond MM, Otto SP, Klausmeier CA (2017) When predators help prey adapt and persist in a changing environment. *American Naturalist* 190: 83–98.

Klausmeier CA, Osmond MM, Kremer CT, Litchman E (2020) Ecological limits to evolutionary rescue. *Phil Trans R Soc B* 375: 20190453

IV. A general framework combining intra- and interspecific trait variation

- Many trait-based theoretical frameworks ignore intraspecific trait variation (adaptive dynamics, ESS maximum approach) or treat it as fixed
- Quantitative genetics can model intraspecific trait variation but typically focuses on a single species



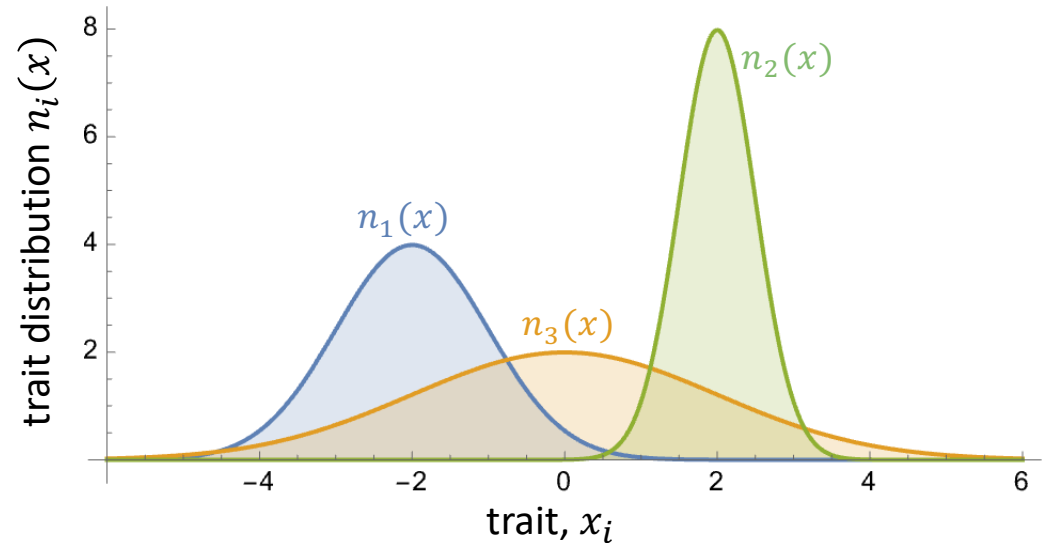
IV. A general framework combining intra- and interspecific trait variation

1. Multi-species moment methods
 - A. Moment dynamical equations
 - B. Invasion criteria / branching conditions
 - C. Example: Lotka-Volterra competition
2. Extension to class-structured populations
 - A. Moment dynamical equations
 - B. Example: two-patch model

(Wickman, Koffel & Klausmeier *Am Nat* 2023)

1. Multi-species Moment Methods

- Consider \mathcal{N} species (\mathcal{N} to be determined) with normally distributed traits
- Each species has a trait distribution $n_i(x)$ characterized by its first three moments
 - 0th – total abundance, N_i
 - 1st – mean trait, x_i
 - 2nd – trait variance, V_i



1. Multi-species Moment Methods

- Individual-level fitness function

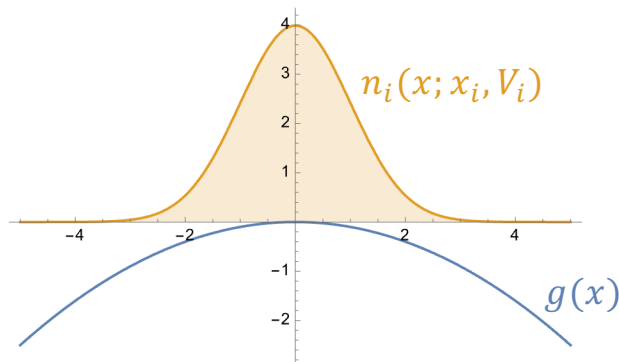
$$g(x; n(\cdot)) = \frac{dn}{ndt}$$

(*N.B.* includes species interactions!)

1. Multi-species Moment Methods

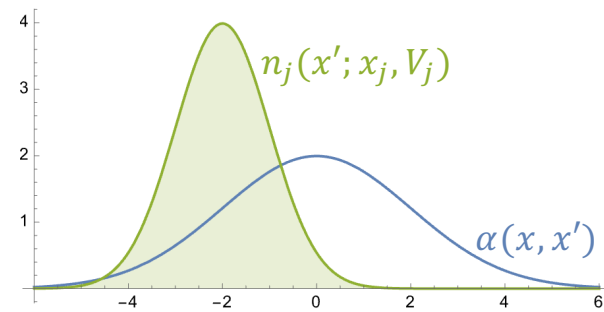
- Fitness and interactions need to be averaged over trait distributions to derive population-level fitness (Gaussian integral)

$$\hat{g}(x_i, V_i)N_i = \int g(x)n_i(x; x_i, V_i)dx$$



$$g(x) = 1 - x^2$$

$$\hat{g}(x_i, V_i) = 1 - x_i^2 - V_i$$



$$\alpha(x, x')n_j(x') = \exp\left(\frac{-(x - x')^2}{2\sigma}\right)n_j(x')$$

$$\hat{\alpha}(x, x_j, V_j) = \exp\left(\frac{-(x-x_j)^2}{2(\sigma+\sqrt{V_j})}\right) \frac{\sigma}{\sqrt{\sigma^2+V_j}}N_j$$

1. Multi-species Moment Methods

Total abundance:

$$\frac{dN_i}{dt} = \hat{g}(x_i, V_i)N_i$$

Trait mean:

$$\frac{dx_i}{dt} = V_i \frac{\partial \hat{g}}{\partial x}(x_i, V_i)$$

Trait variance:

$$\frac{dV_i}{dt} = V_i^2 \frac{\partial^2 \hat{g}}{\partial x^2}(x_i, V_i) + \hat{b}(x_i, V_i)M$$

(\hat{b} is birth rate, M is mutation variance)

Example: Lotka-Volterra Competition

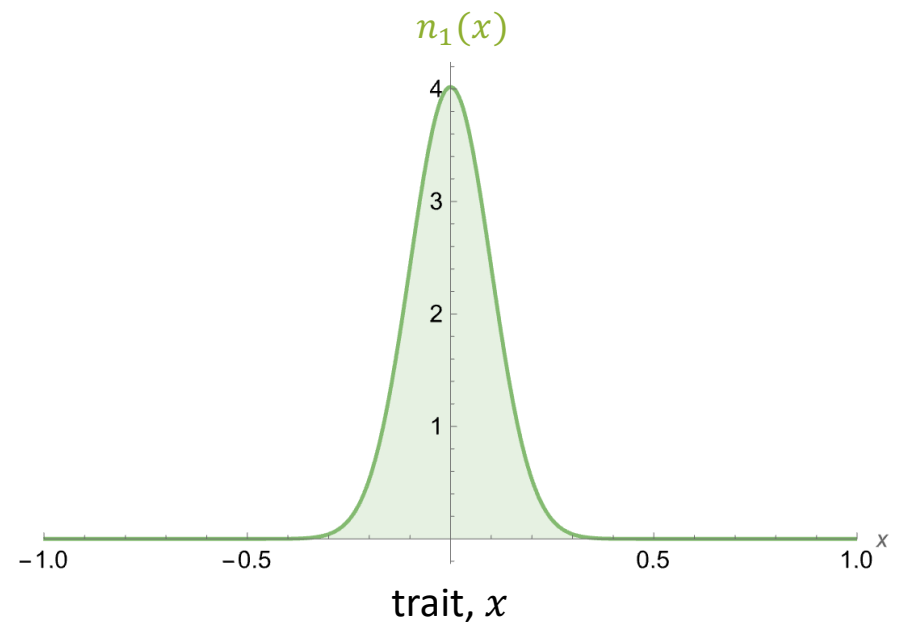
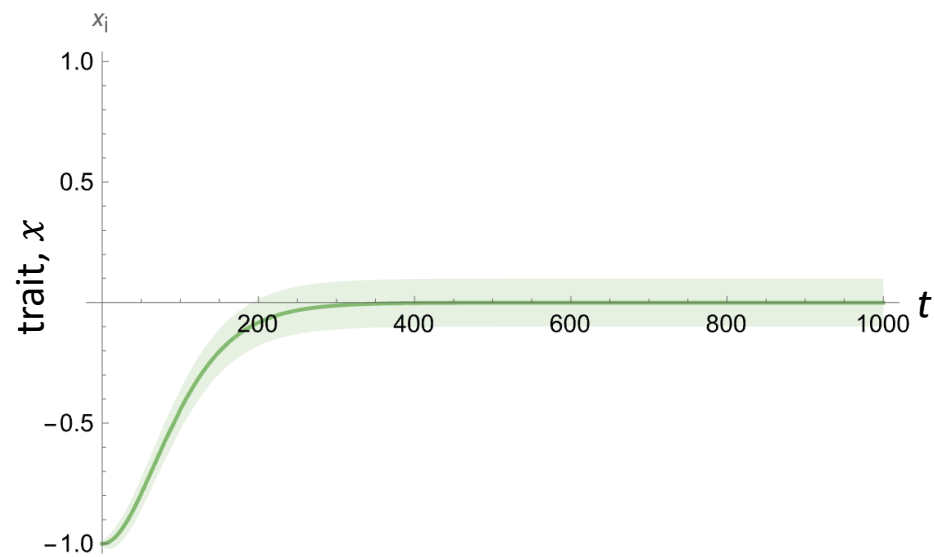
$$\frac{dN_i}{dt} = \left(1 - x_i^2 - V_i - \sum_{j=1}^{\mathcal{N}} \frac{\sigma}{\sqrt{\sigma^2 + V_i + V_j}} \exp(-(x_i - x_j)^2 / (2(\sigma^2 + V_i + V_j))) N_j \right) N_i$$

$$\frac{dx_i}{dt} = V_i \left(-2x_i + \sum_{j=1}^{\mathcal{N}} \frac{\sigma(x_i - x_j)}{(\sigma^2 + V_i + V_j)^{3/2}} \exp(-(x_i - x_j)^2 / (2(\sigma^2 + V_i + V_j))) N_j \right)$$

$$\frac{dV_i}{dt} = V_i^2 \left(-2 + \sum_{j=1}^{\mathcal{N}} \frac{\sigma(\sigma^2 + V_i + V_j - (x_i - x_j)^2)}{(\sigma^2 + V_i + V_j)^{5/2}} \exp(-(x_i - x_j)^2 / (2(\sigma^2 + V_i + V_j))) N_j \right) + M$$

Example: Lotka-Volterra Competition

$$(\mathcal{N} = 1, \sigma = 1, M = 10^{-4})$$

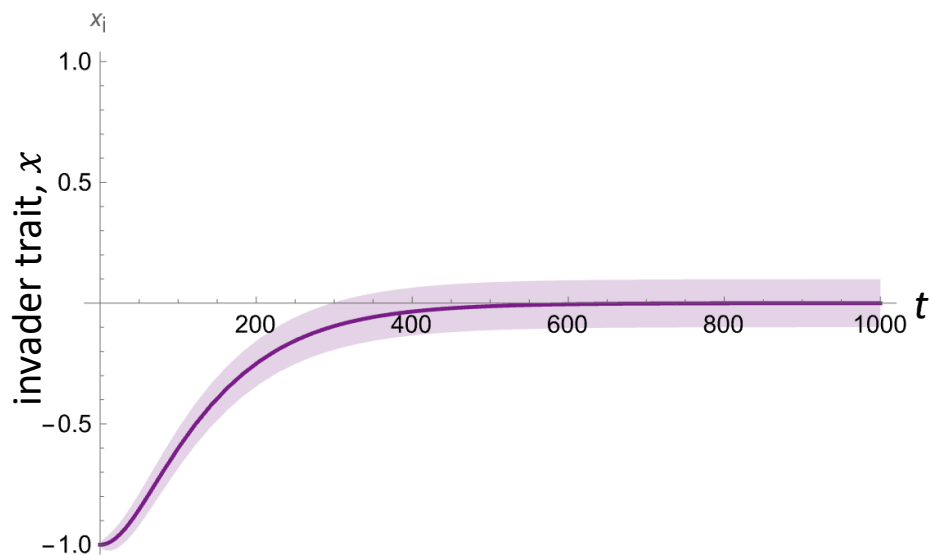


1.B. Invasion Criteria in Moment-Structured Populations

- To determine the evolutionarily stable community, we need invasion criteria
- 💡 Idea: introduce rare invader ($N_0 \approx 0$), evolve its trait mean, x_0 , and trait variance, V_0 , in the environment set by resident(s) until it reaches a stable equilibrium (x_0^*, V_0^*) , then calculate its population growth rate, $\hat{g}(x_0^*, V_0^*)$
- Can be visualize with phase-plane

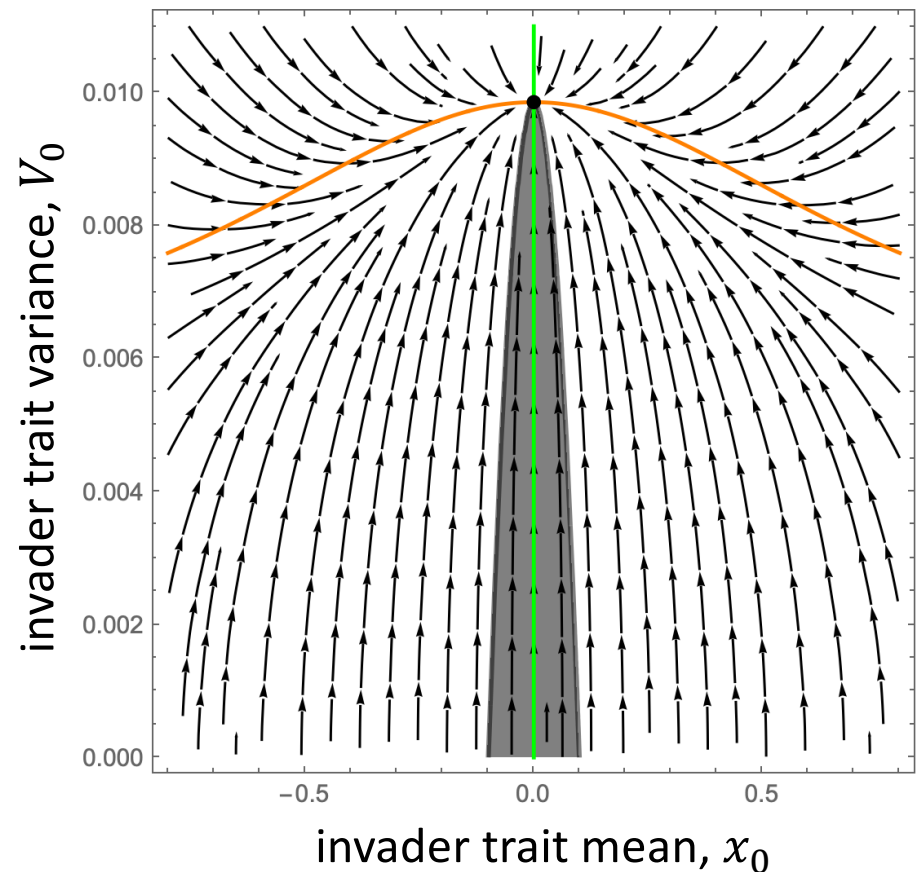
Example: Lotka-Volterra Competition

$$(\mathcal{N} = 1, \sigma = 1, M = 10^{-4})$$



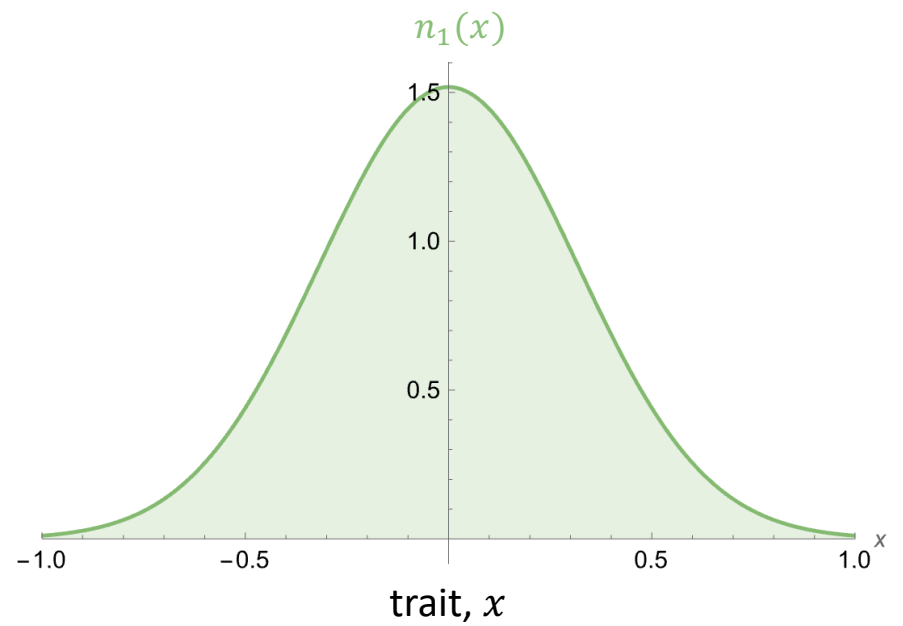
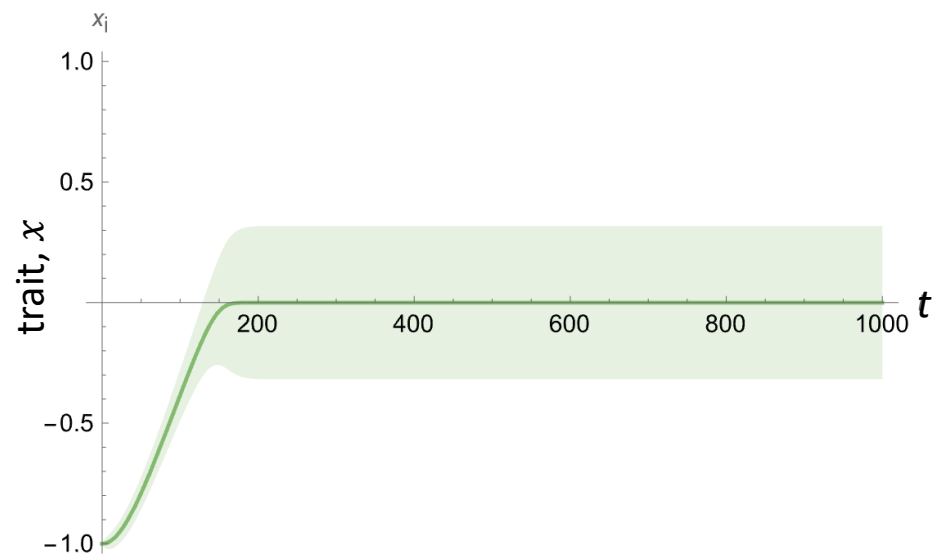
Convergence to resident, then neutral invasion rate ($\hat{g}(x_0^*, V_0^*) = 0$) \Rightarrow failed invasion

\Rightarrow one-species evolutionarily stable community



Example: Lotka-Volterra Competition

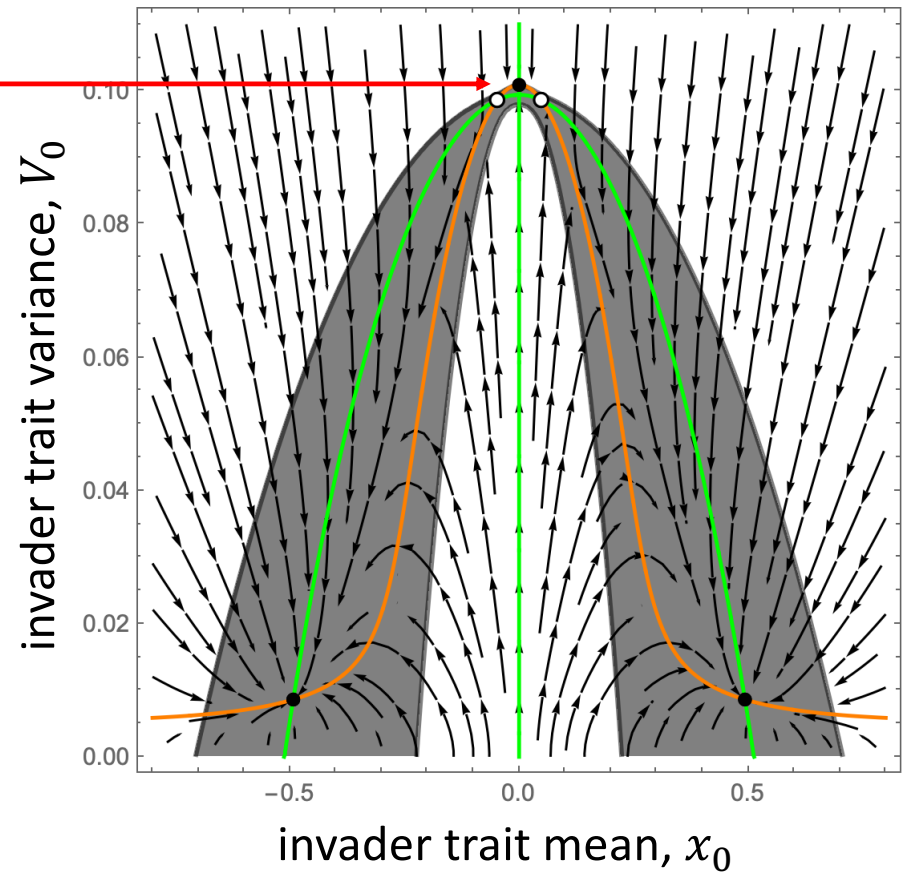
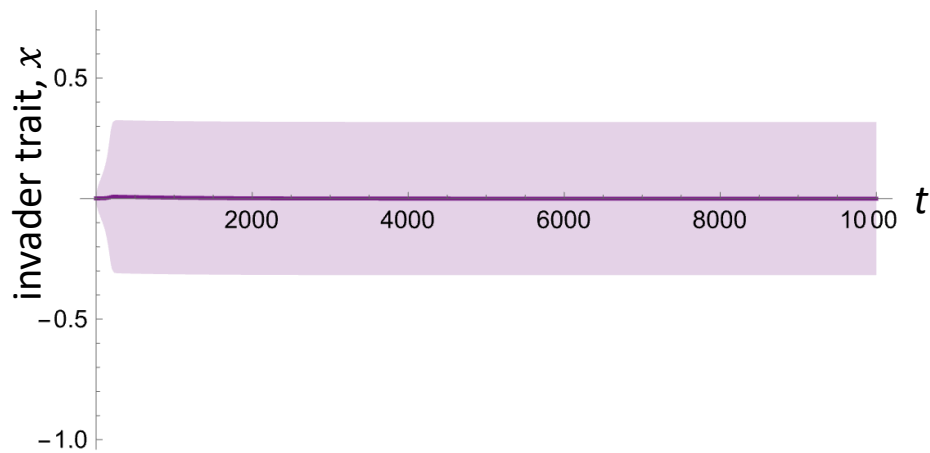
$$(\mathcal{N} = 1, \sigma = 0.5, M = 10^{-4})$$



Example: Lotka-Volterra Competition

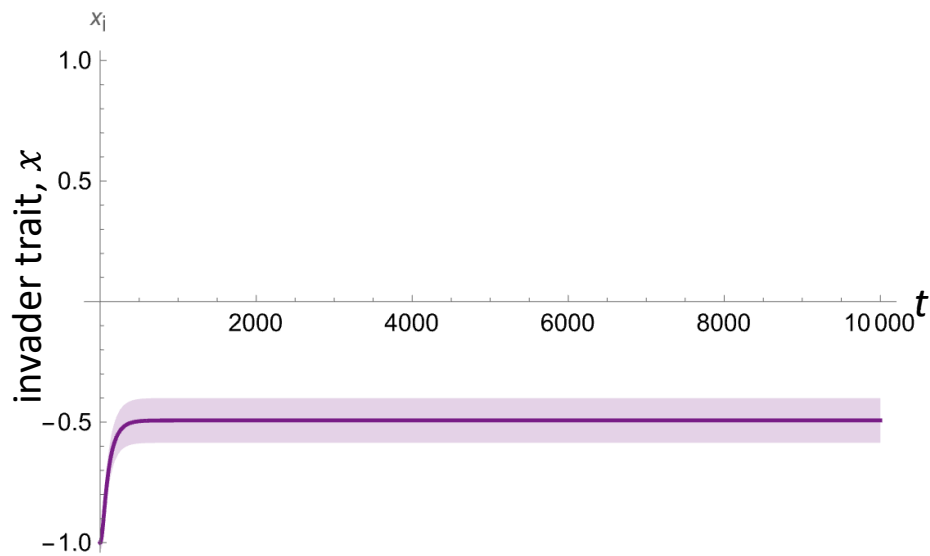
$$(\mathcal{N} = 1, \sigma = 0.5, M = 10^{-4})$$

Convergence to resident, then neutral invasion rate ($\hat{g}(x_0^*, V_0^*)=0$) \Rightarrow failed invasion



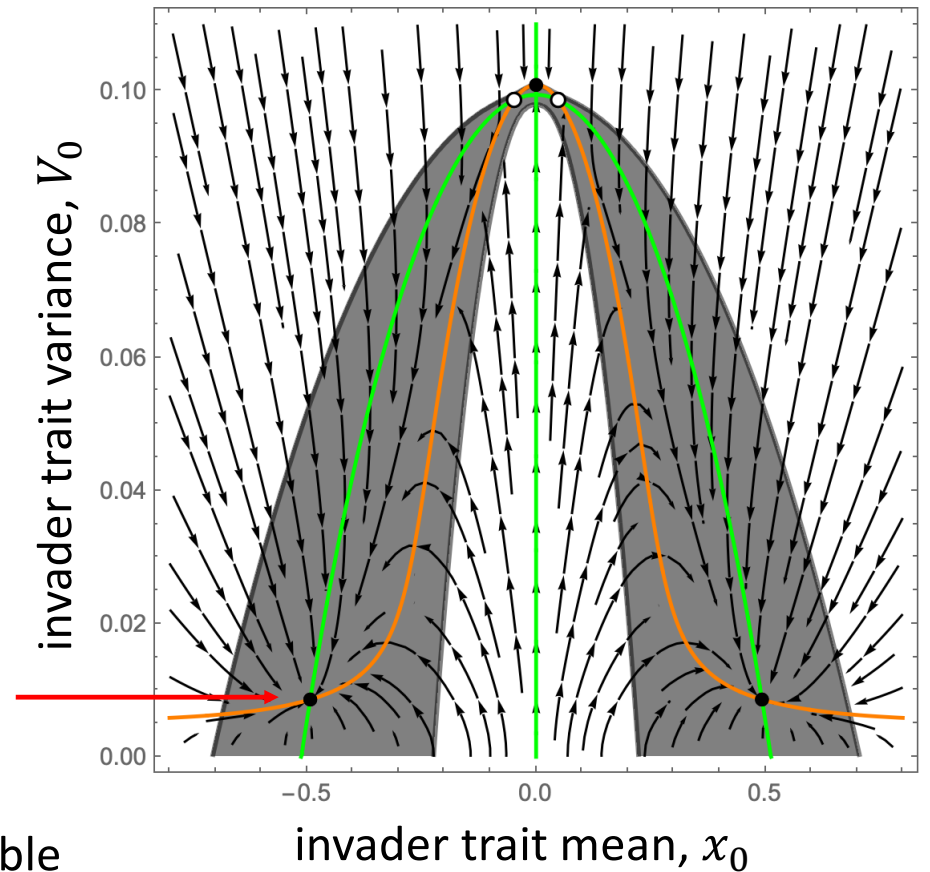
Example: Lotka-Volterra Competition

$$(\mathcal{N} = 1, \sigma = 0.5, M = 10^{-4})$$



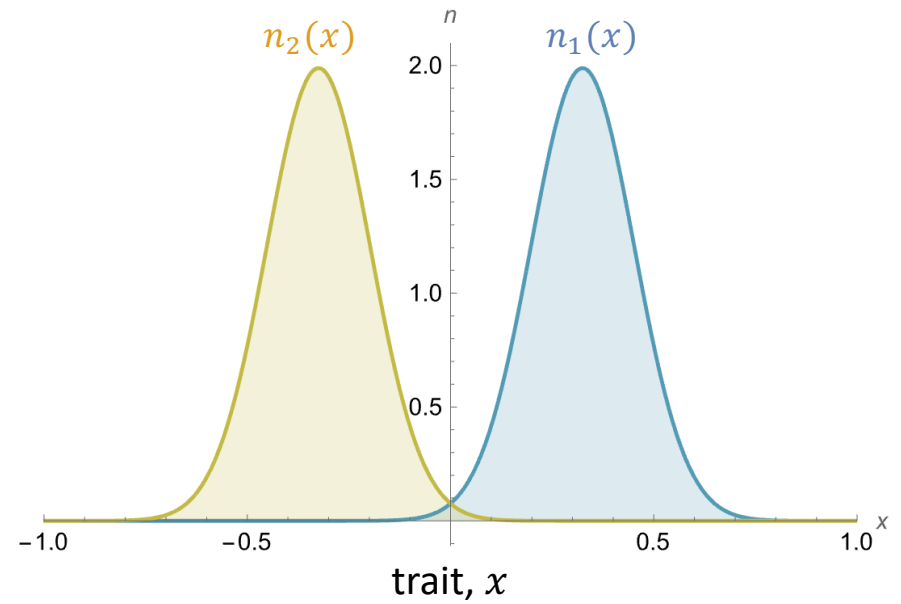
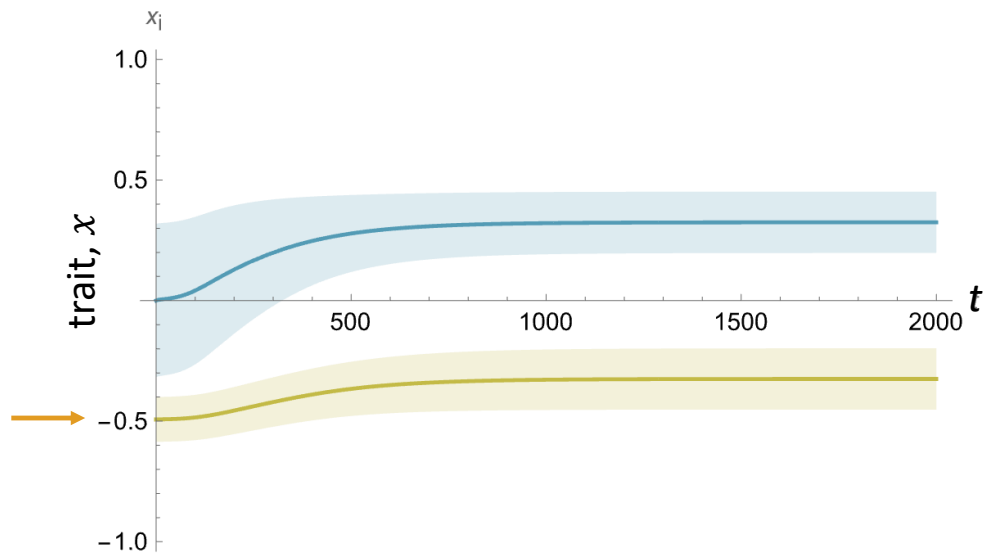
Distinct equilibrium moments, then positive invasion rate ($\hat{g}(x_0^*, V_0^*) = 0.0295$)
 \Rightarrow successful invasion

\Rightarrow locally evolutionarily stable, but globally invasible



Example: Lotka-Volterra Competition

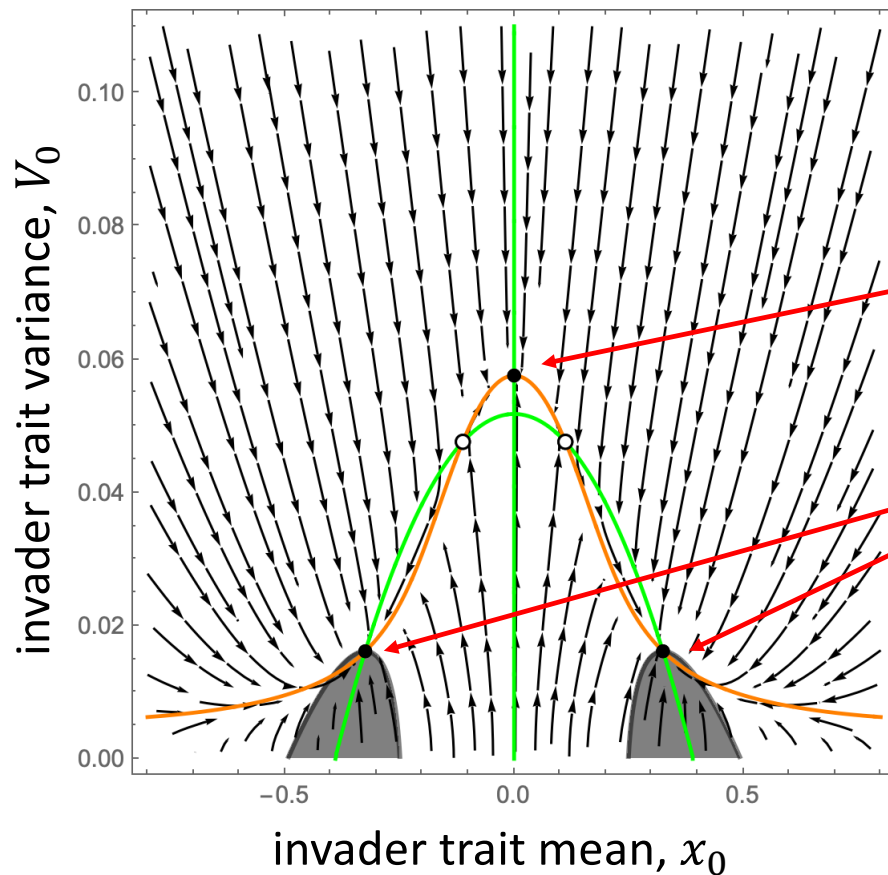
$$(\mathcal{N} = 2, \sigma = 0.5, M = 10^{-4})$$



After invasion, two species coexist

Example: Lotka-Volterra Competition

$$(\mathcal{N} = 2, \sigma = 0.5, M = 10^{-4})$$



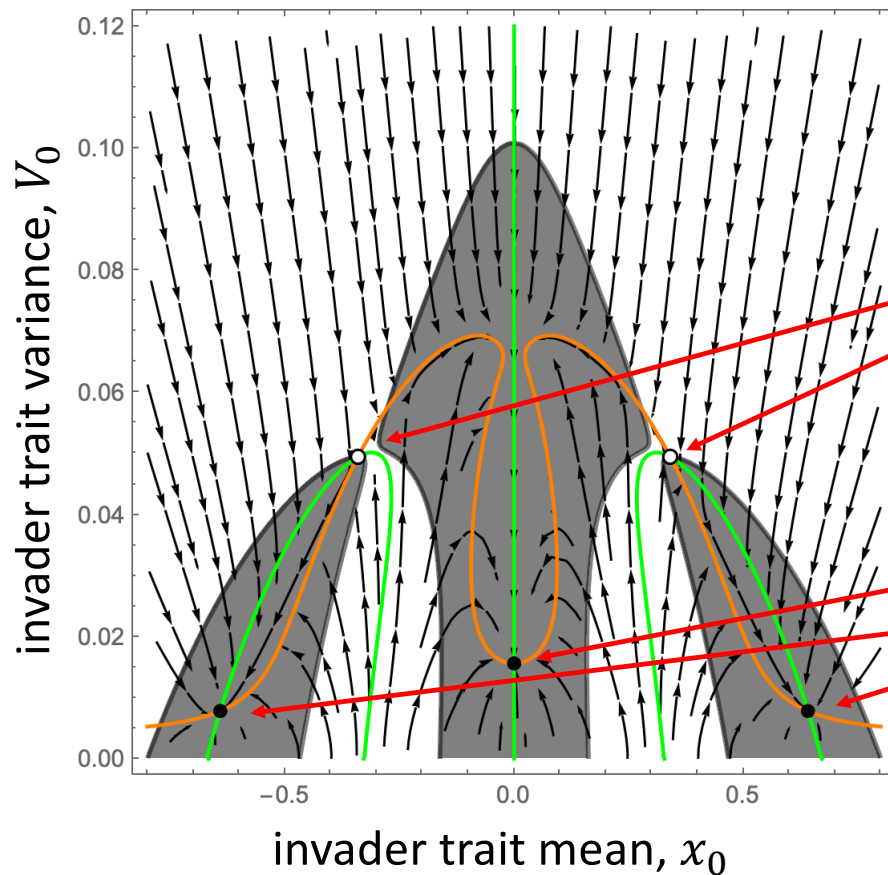
Convergence to another equilibrium,
then negative invasion rate
 $(\hat{g}(x_0^*, V_0^*) < 0) \Rightarrow$ failed invasion

Convergence to resident, then neutral
invasion rate
 $(\hat{g}(x_0^*, V_0^* = 0) = 0) \Rightarrow$ failed invasion

\Rightarrow two-species evolutionarily stable
community

Example: Lotka-Volterra Competition

$$(\mathcal{N} = 2, \sigma = 0.3, M = 10^{-4})$$

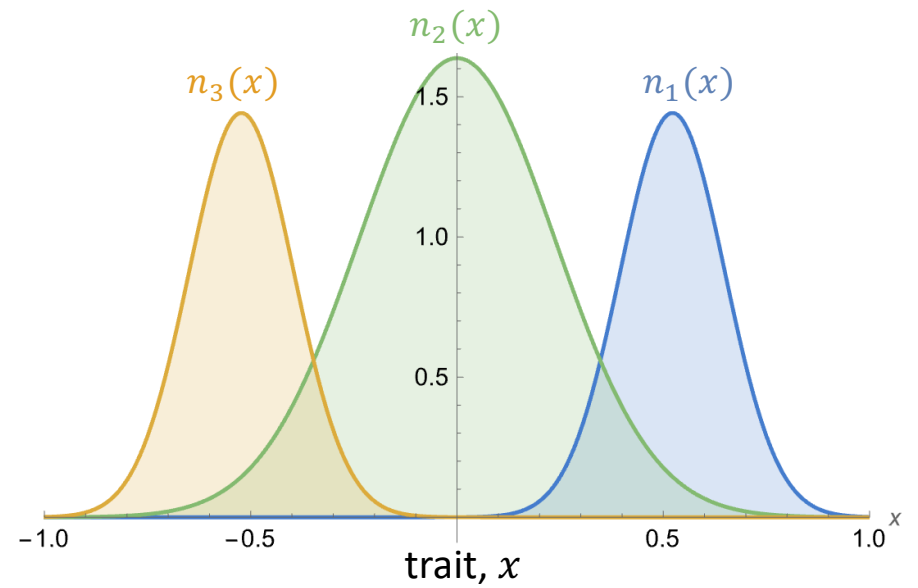
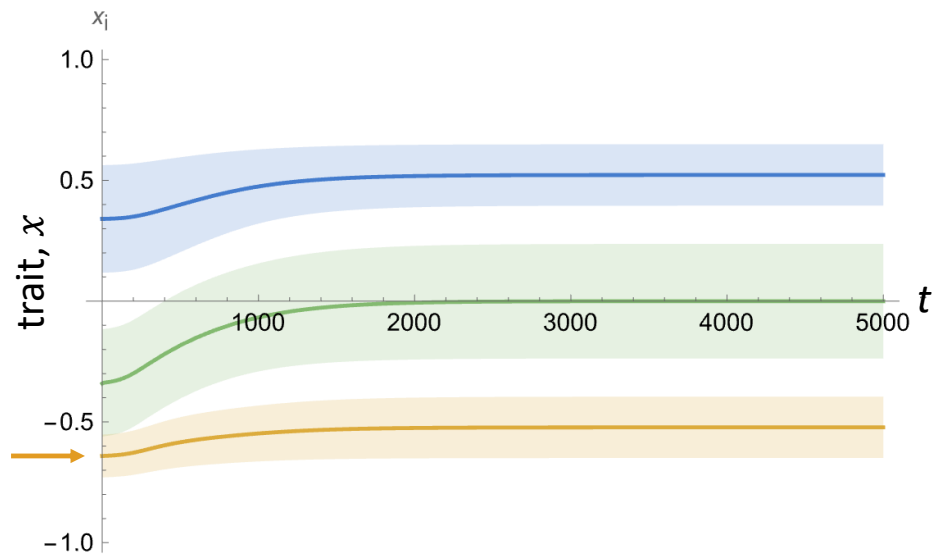


Two residents are not convergence stable \Rightarrow branching point

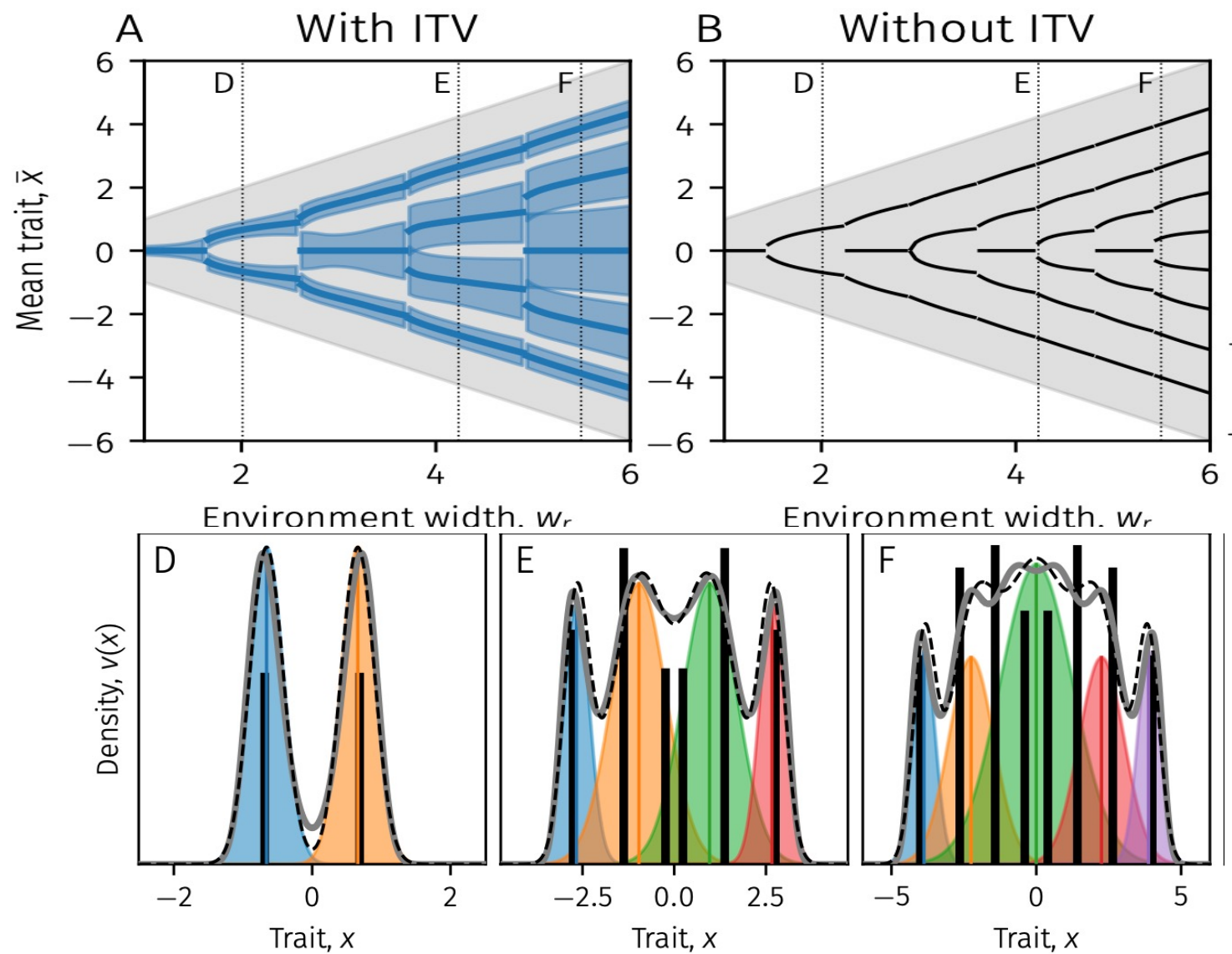
Three invader equilibria, each with positive invasion rate ($\hat{g}(x_0^*, V_0^*) > 0$)

Example: Lotka-Volterra Competition

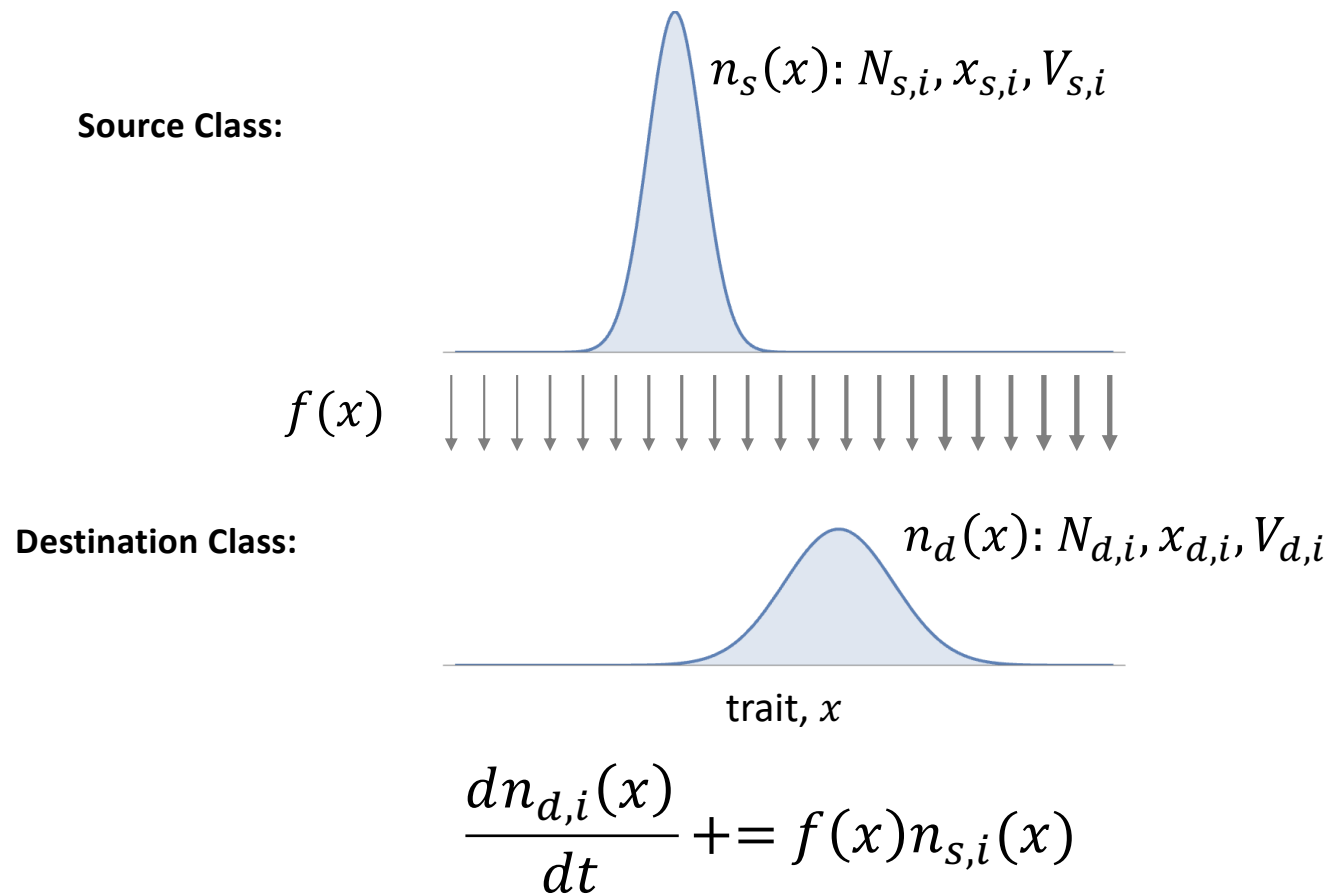
$$(\mathcal{N} = 3, \sigma = 0.3, M = 10^{-4})$$



After invasion, three species coexist.



2. Extension to Class-Structured Populations



2. Extension to Class-Structured Populations

Total abundance:

$$\frac{dN_{d,i}}{dt} += \hat{f}(x_{s,i}, V_{s,i})N_{s,i}$$
$$\begin{bmatrix} \text{change in} \\ \text{destination} \\ \text{abundance} \end{bmatrix} += \begin{bmatrix} \text{population-level} \\ \text{rate} \end{bmatrix} \begin{bmatrix} \text{source} \\ \text{abundance} \end{bmatrix}$$

2. Extension to Class-Structured Populations

Trait mean:

$$\frac{dx_{d,i}}{dt} + = \frac{N_{s,i}}{N_{d,i}} \left(V_{s,i} \frac{\partial \hat{f}}{\partial x} (x_{s,i}, V_{s,i}) + \hat{f}(x_{s,i}, V_{s,i})(x_{s,i} - x_{d,i}) \right)$$

$$\begin{bmatrix} \text{change in} \\ \text{destination} \\ \text{trait-mean} \end{bmatrix} + = \begin{bmatrix} \text{relative} \\ \text{abundance} \end{bmatrix} \times \left(\begin{bmatrix} \text{directional} \\ \text{selection} \end{bmatrix} + \begin{bmatrix} \text{trait-mean} \\ \text{flow} \end{bmatrix} \right)$$

2. Extension to Class-Structured Populations

Trait variance:

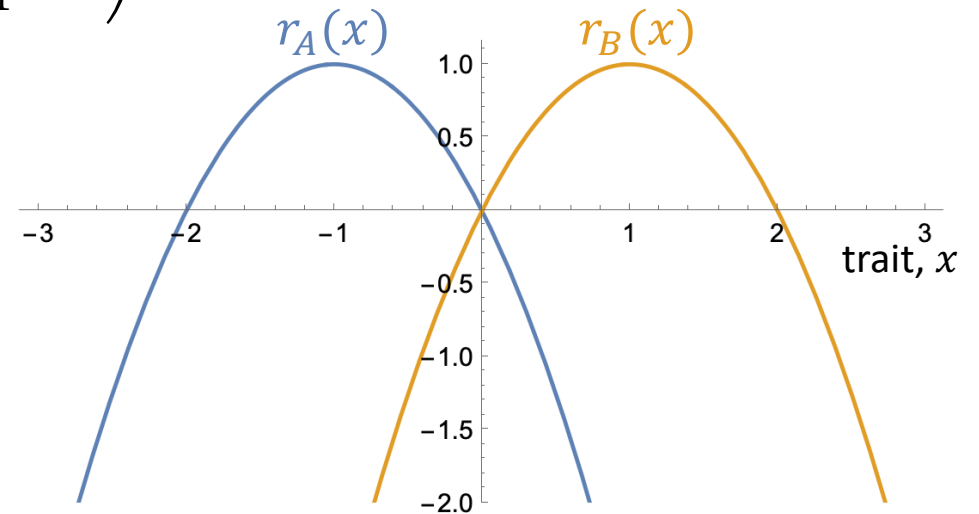
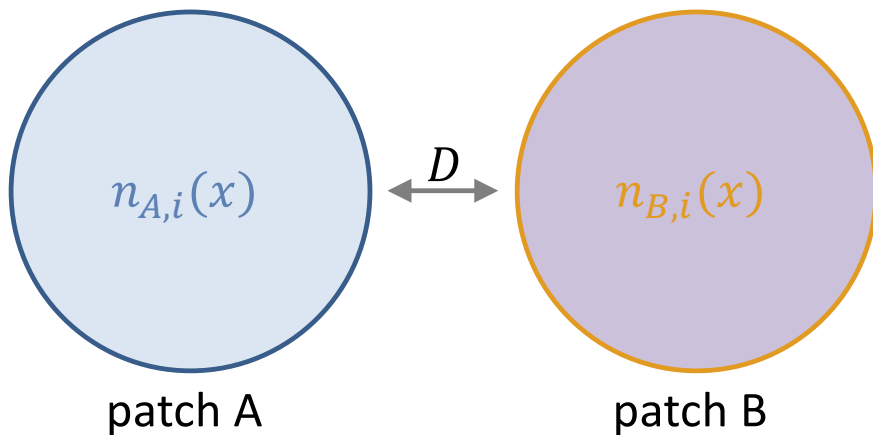
$$\frac{dV_{d,i}}{dt} += \frac{N_{s,i}}{N_{d,i}} \left(V_{s,i}^2 \frac{\partial^2 \hat{f}}{\partial x^2}(x_{s,i}, V_{s,i}) + \hat{f}(x_{s,i}, V_{s,i})(V_{s,i} - V_{d,i}) + \hat{f}(x_{s,i}, V_{s,i})(x_{s,i} - x_{d,i})^2 \right. \\ \left. + 2V_{s,i} \frac{\partial \hat{f}}{\partial x}(x_{s,i}, V_{s,i})(x_{s,i} - x_{d,i}) + \hat{f}(x_{s,i}, V_{s,i})M \right)$$

$$\left[\begin{array}{c} \text{change in} \\ \text{destination} \\ \text{trait-variance} \end{array} \right] += \left[\begin{array}{c} \text{relative} \\ \text{abundance} \end{array} \right] \times$$

$$\left(\left[\begin{array}{c} \text{quadratic} \\ \text{selection} \end{array} \right] + \left[\begin{array}{c} \text{trait-variance} \\ \text{flow} \end{array} \right] + \left[\begin{array}{c} \text{between-to-} \\ \text{within-class} \\ \text{variance flow} \end{array} \right] + \left[\begin{array}{c} \text{directional-selection} \times \\ \text{trait-mean interaction} \end{array} \right] + \left[\text{mutation} \right] \right)$$

Example: Two-Patch Model

$$\frac{dn_{A,i}(x)}{dt} = \left(r_A(x) - \sum_{j=1}^{\mathcal{N}} n_{A,j} \right) n_{A,i} + D(n_{B,i} - n_{A,i})$$
$$\frac{dn_{B,i}(x)}{dt} = \left(r_B(x) - \sum_{j=1}^{\mathcal{N}} n_{B,j} \right) n_{B,i} + D(n_{A,i} - n_{B,i})$$



Example: Two-Patch Model

Total Abundance:

$$\frac{dN_{A,i}}{dt} = \left(1 - (x_{A,i} - x_A^*)^2 - V_{A,i} - \sum_{j=1}^{\mathcal{N}} N_{A,j} \right) N_{A,i} + D(N_{B,i} - N_{A,i})$$
$$\frac{dN_{B,i}}{dt} = \left(1 - (x_{B,i} - x_B^*)^2 - V_{B,i} - \sum_{j=1}^{\mathcal{N}} N_{B,j} \right) N_{B,i} + D(N_{A,i} - N_{B,i})$$

Trait Mean:

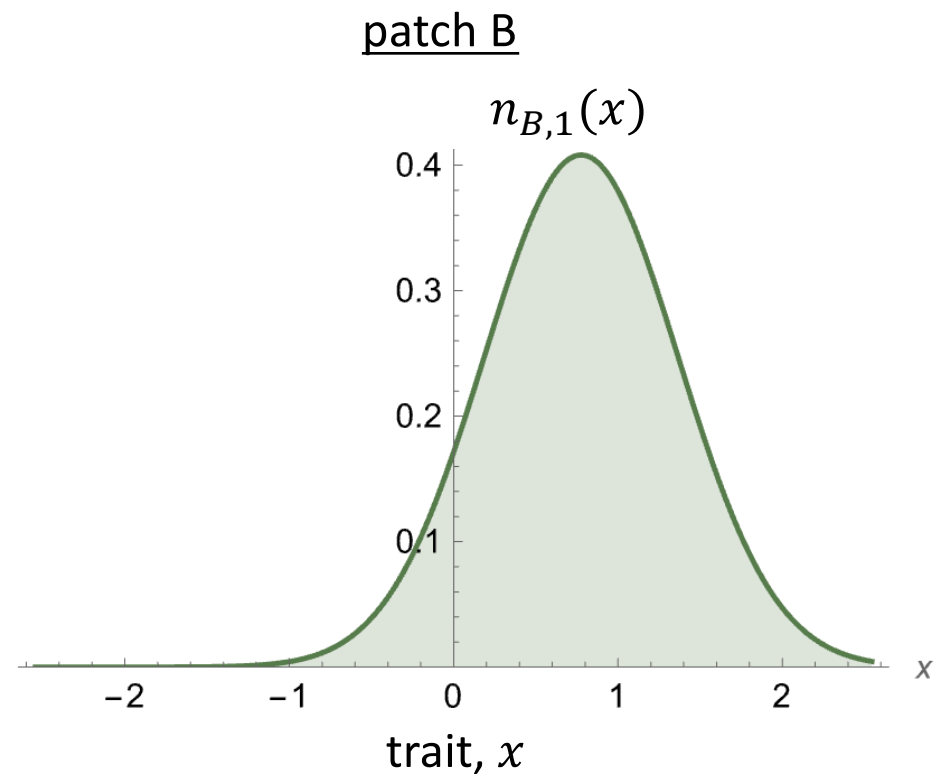
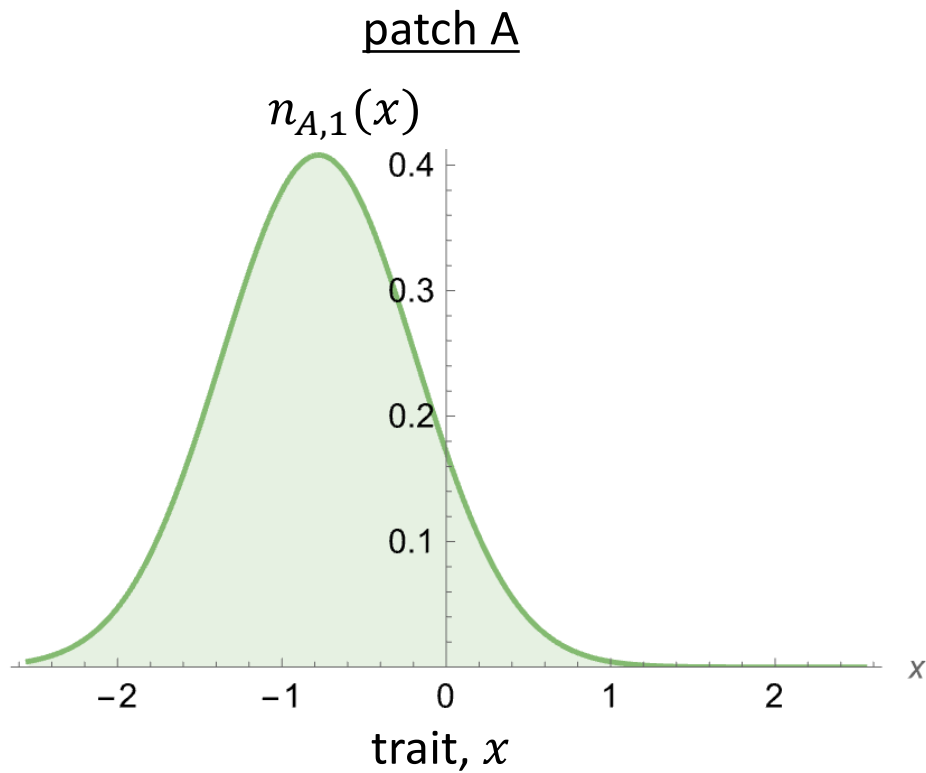
$$\frac{dx_{A,i}}{dt} = -2V_{A,i}(x_{A,i} - x_A^*) + D \frac{N_{B,i}}{N_{A,i}}(x_{B,i} - x_{A,i})$$
$$\frac{dx_{B,i}}{dt} = -2V_{B,i}(x_{B,i} - x_B^*) + D \frac{N_{A,i}}{N_{B,i}}(x_{A,i} - x_{B,i})$$

Trait Variance:

$$\frac{dV_{A,i}}{dt} = M - 2V_{A,i}^2 + D \frac{N_{B,i}}{N_{A,i}}(V_{B,i} - V_{A,i} + (x_{B,i} - x_{A,i})^2)$$
$$\frac{dV_{B,i}}{dt} = M - 2V_{B,i}^2 + D \frac{N_{A,i}}{N_{B,i}}(V_{A,i} - V_{B,i} + (x_{A,i} - x_{B,i})^2)$$

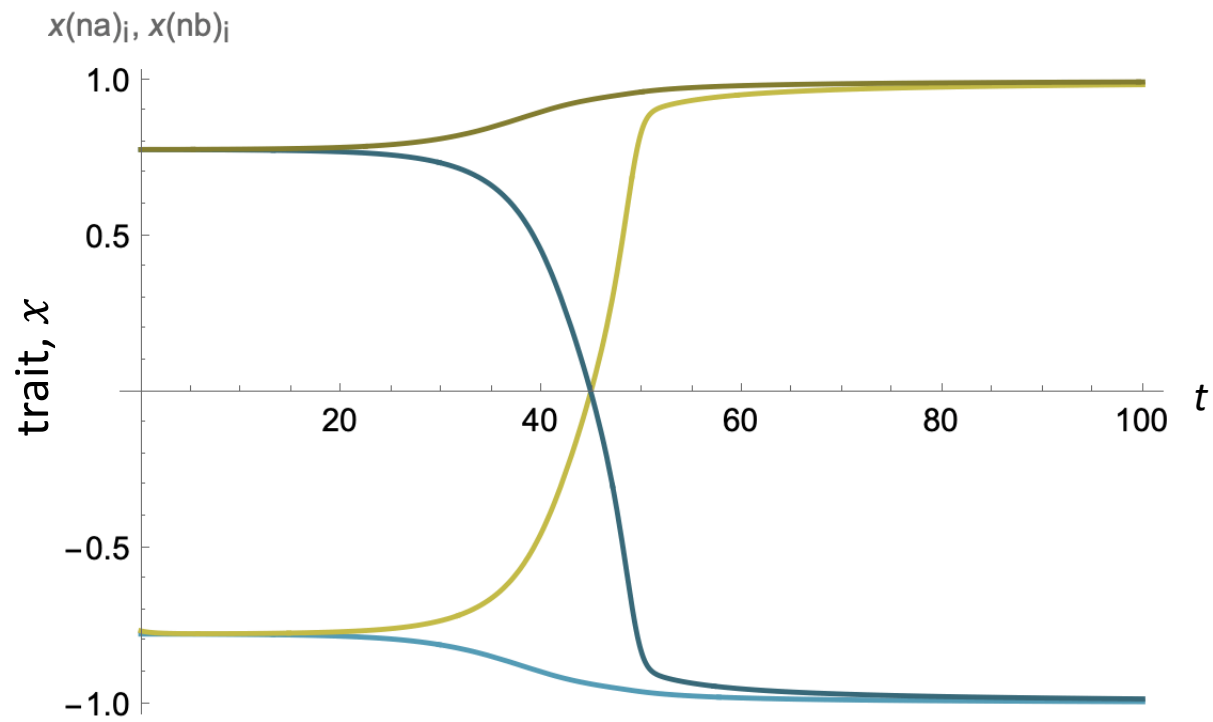
Example: Two-Patch Model

$(\mathcal{N} = 1, D = 0.1, M = 0)$



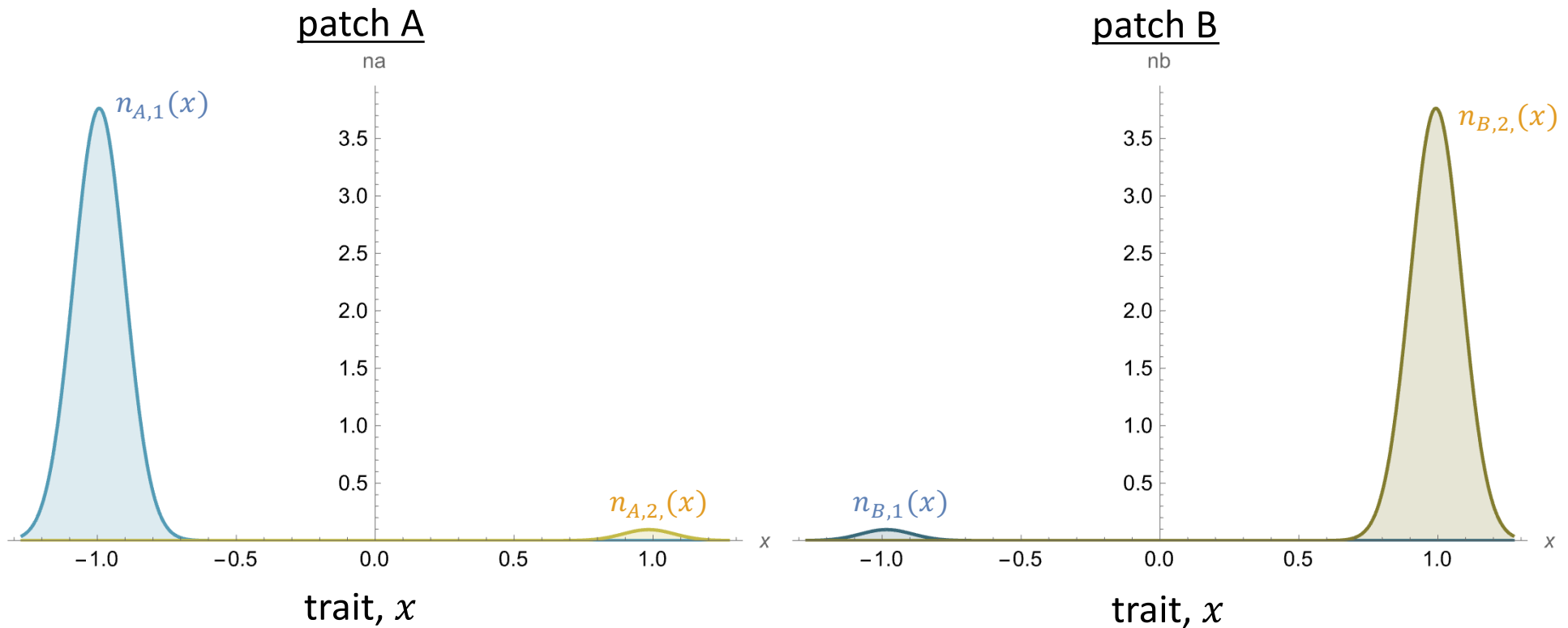
Example: Two-Patch Model

$(\mathcal{N} = 2, D = 0.1, M = 0)$



Example: Two-Patch Model

($\mathcal{N} = 2, D = 0.1, M = 0$)



IV. Conclusions

- Multi-species moment equations provide an efficient and intuitive way to model eco-evolutionary dynamics, including the causes and consequences of intraspecific trait variation
- Invasion criteria can be calculated by evolving the trait mean and variance of a rare invader, along with branching point conditions
- Intraspecific trait variation decreases species richness
- Spatial models can result in local adaptation and species sorting in heterogeneous environments
- See also: Lion S, Boots M, Sasaki A. 2022. Multimorph eco-evolutionary dynamics in structured populations. *American Naturalist* 200: 345–372

(Wickman, Koffel & Klausmeier *Am Nat* 2023)

Overall Conclusions

- Diversity is an essential feature of complex systems such as ecological communities
- Trait-based eco-evolutionary modeling is a mature field that provides tools to understand the origin & maintenance of diversity
- Diversity is key to understanding ecological resilience

Competitive communities

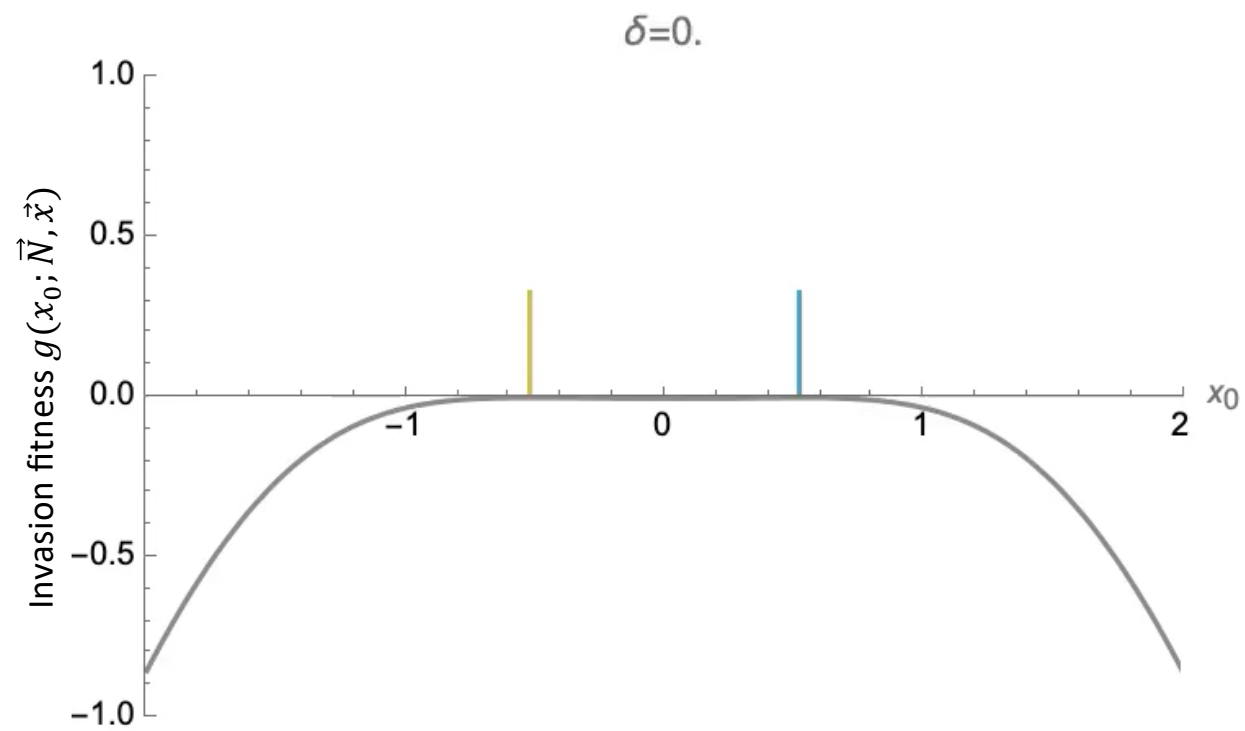
Evolutionary quantitative genetics framework:

$$\begin{aligned}\frac{dN_i}{dt} &= g(x_i; \vec{N}, \vec{x}) N_i \\ \frac{dx_i}{dt} &= V \frac{\partial g}{\partial x}(x_i; \vec{N}, \vec{x}) - \delta\end{aligned}$$

Trait-based Lotka-Volterra competition model:

$$g(x_i; \vec{N}, \vec{x}) = r(x_i) - \sum_{j=1}^{\mathcal{N}} \alpha(x_i, x_j) N_j$$

Competitive communities



Competitive communities

