QUESTION 1: Recursive Functions

For the following functions, (1) define the base case when y=0 [**a,c**] and x=0 [**b**] and (2) define the relationship between y and y+1 [eg. y+1 = what function or operation of y? for **a,c,d,e**] and between x and x+1 [**b**]. You can assume you know how to find the values on the right side of each equation, so the question is specifically how to increment these operations for y+1 and x+1.

(a) exp(x, y) = x^y

(b) pred(x) = x - 1

(c) sub(x, y) = x - y

(d) min(x, y)

(e) max(x, y)

You may use the functions defined within the lecture as well as any functions defined here within your solutions:

add(x, y) = x + y

mult(x, y) = x * y

NOTE: Do not worry about negative values; assume any negative solutions return zero.

## QUESTION 2: Turing Machines

Design a Turing machine that uses a tape alphabet {•,0,1} to calculate the successor function (x+1). The machine starts in the state shown in the top depiction and should end in the state shown in the bottom depiction in the diagram below.

(1) Define the functions CARRY, RETURN, HALT and (2) describe the sequence of how they would be implemented by your machine when calculating the successor function on the tape in the diagram. CARRY and RETURN function should be defined for the three possible conditions that the machine might read on the tape {•,0,1}. The HALT function needs only to be defined for the decimal • state. You can assume that a "move" function exists and that negative move

values indicate leftward movement and positive move values indicate rightward movement, *eg.* "–3" would move the cursor 3 indices to the left.

When defining the functions, you may use a mathematic notation or describe these in natural language, *eg.* "If in state •, RETURN does..." or "*F* (RETURN, •) = ..."
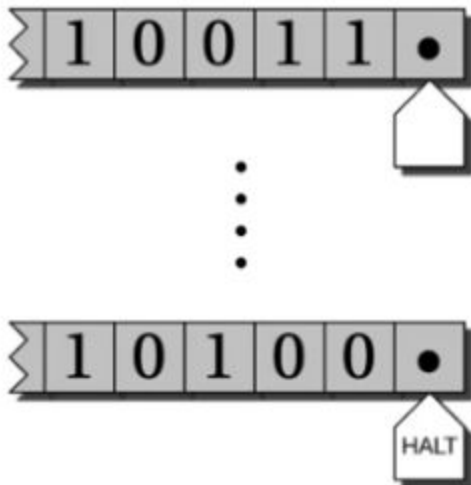
A reminder about counting in binary: The integers shown in the question are 19 (top) and 20 (bottom). Binary increments in the following way: 0 + 1 = 1, 1 + 1 = 0 with the 1 carried to the next position (to the left). For example, the successor of 001 is 010, the successor of 010 is 011, and the successor of 011 is 100.

The questions correspond to the Recursive Functions (QUESTION 1) and Turing Machines (QUESTION 2) Quizzes, where you can find more information.

Download the video quizzes here.