Determining Abstract Computational Machines using Causal Discovery

Group # causality-for-computation

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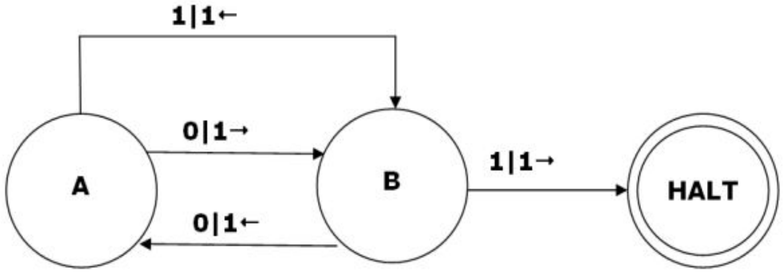
Theory of Computation

What is the mechanism underlying any given process?

- What is a mechanism? ***A definite procedure***
- How to formalize a definite procedure? ***An abstract computational machine***



- What is the abstract computational machine underlying natural processes? ***An epsilon-machine***

Crutchfield, J. P. (1994). The calculi of emergence: computation, dynamics and induction. *Physica D: Nonlinear Phenomena*, *75*(1-3), 11-54.

Computational Mechanics

Example: Fair Coin

Process

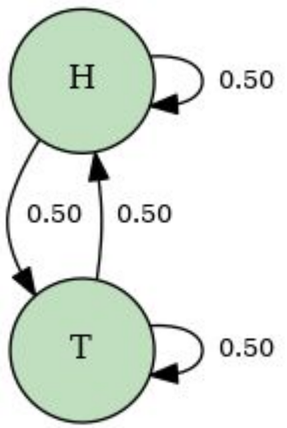
…HTHTHTHT…

…HTHHTTHT…

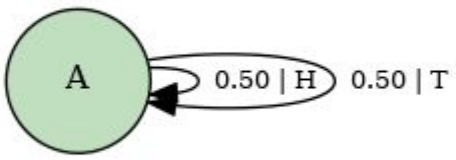
…HHHHHHHH…

…TTTTTTTT…

An abstract machine



Epsilon-machine



Causal Equivalence Principle

$$P(\overrightarrow{S} | \overleftarrow{S}_t) = P(\overrightarrow{S} | \overleftarrow{S}_{t'}) \iff \overleftarrow{S}_t \sim \overleftarrow{S}_{t'}$$

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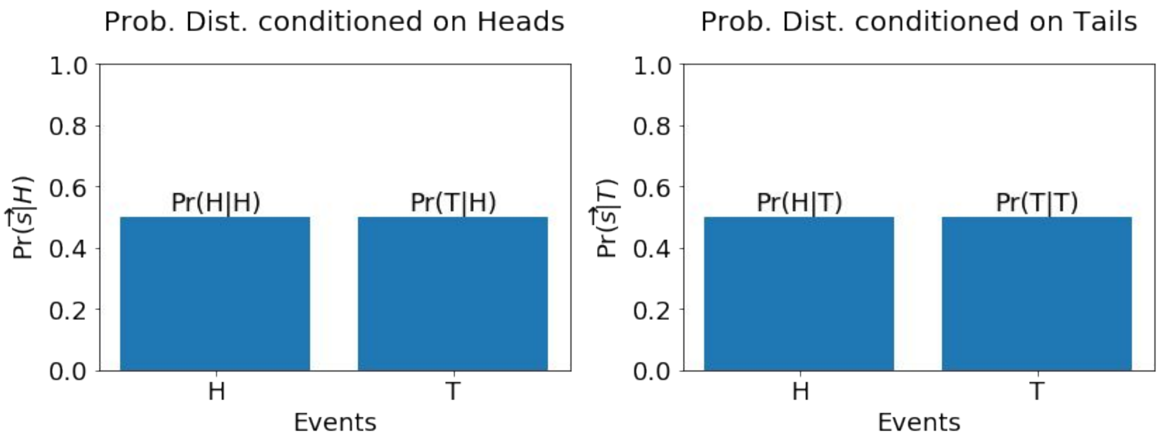

Heads and Tails belong to the ***same causal state*** because conditioning on them leads to the ***same probability distributions*** over the future

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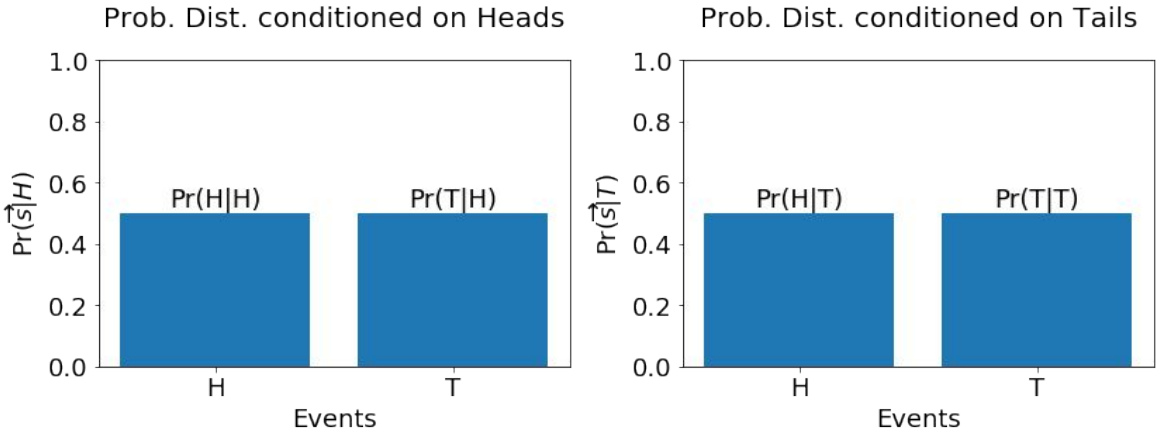

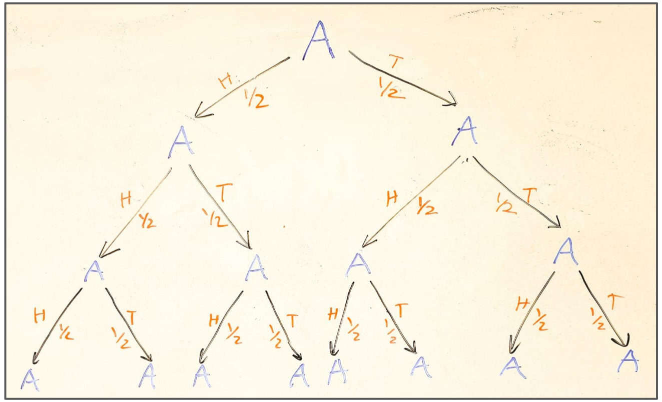

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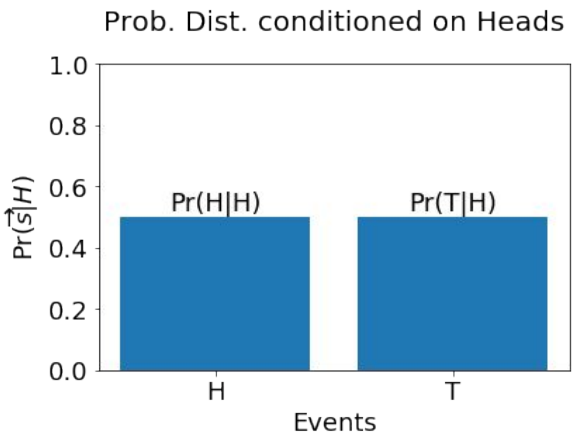

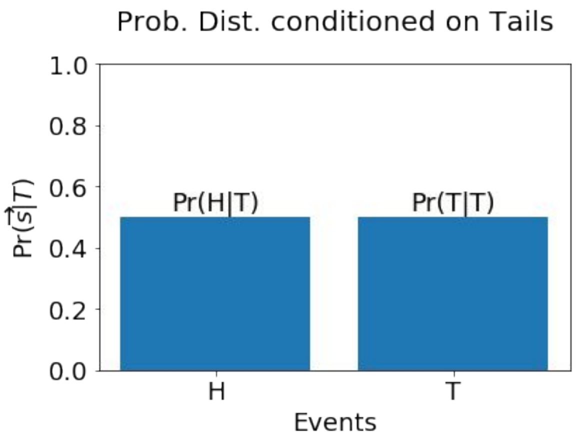

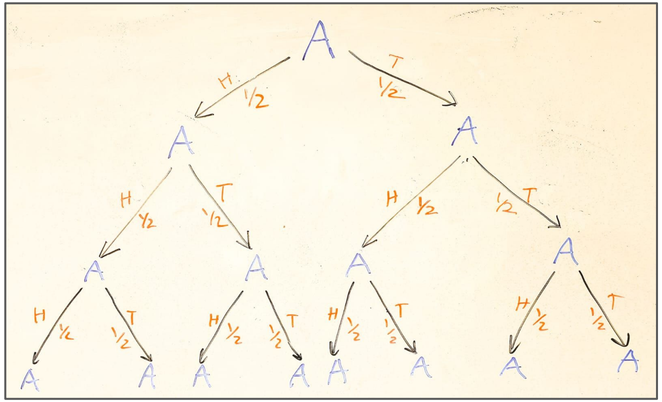

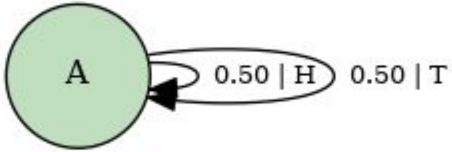

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Current Abstract Machines: Unanswered Questions

- ➔ What would happen if there are 2 or more variables in the given system?
- ➔ Are causal states always determined by equivalence of conditional probabilities?

Approach: Use ideas from ***time series based causal inference***

Kathpalia, A., & Nagaraj, N. (2021). Measuring causality: The science of cause and effect. *Resonance*, *26*, 191-210..

Coupled Coins

Past of Coin Y influences future of coin X :

These two coins are fair except one condition :

if Y(t) = H then X(t+1) = T

t :	0	1	2	3	4	5	6	7	8	9	10	
X :	H	T	T	T	H	H	T	T	T	T	H	...
Y :	H	T	H	T	T	H	T	H	H	T	T	...

Coupled Coin: X and Y dynamics as a single process

We can represent state of the system

- as a vector, eg; [H, T]
- construct a causal tree

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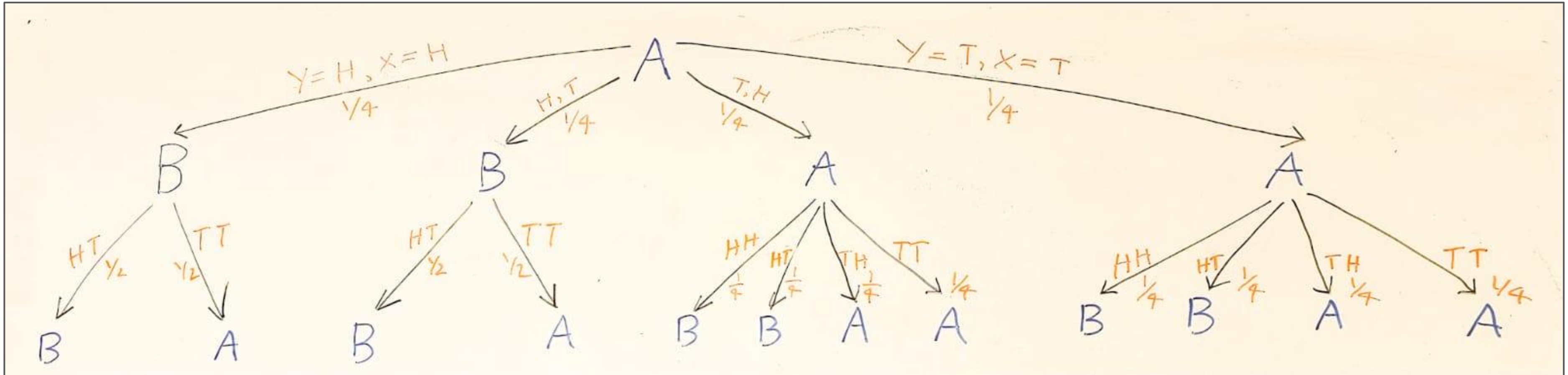

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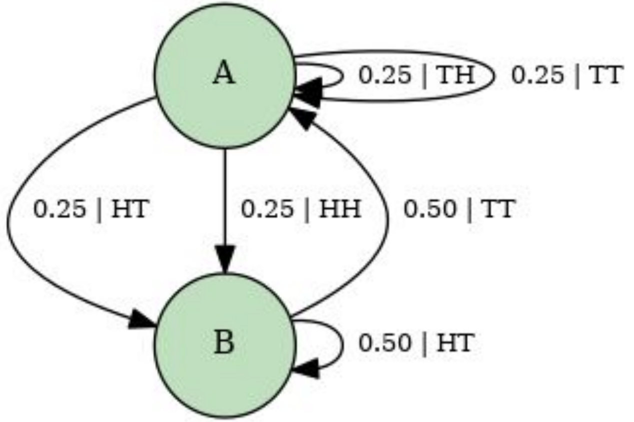
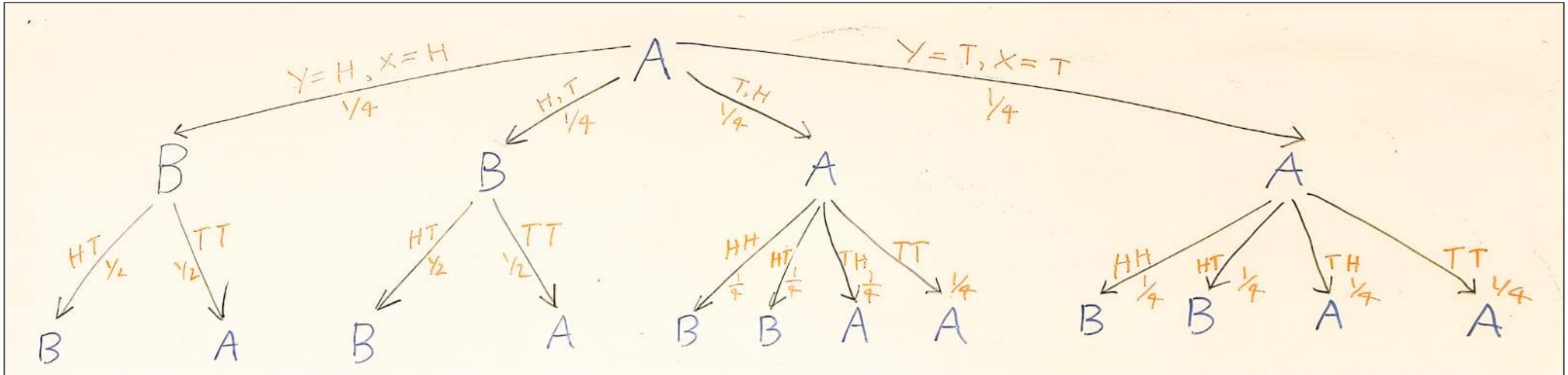
[X,Y] :

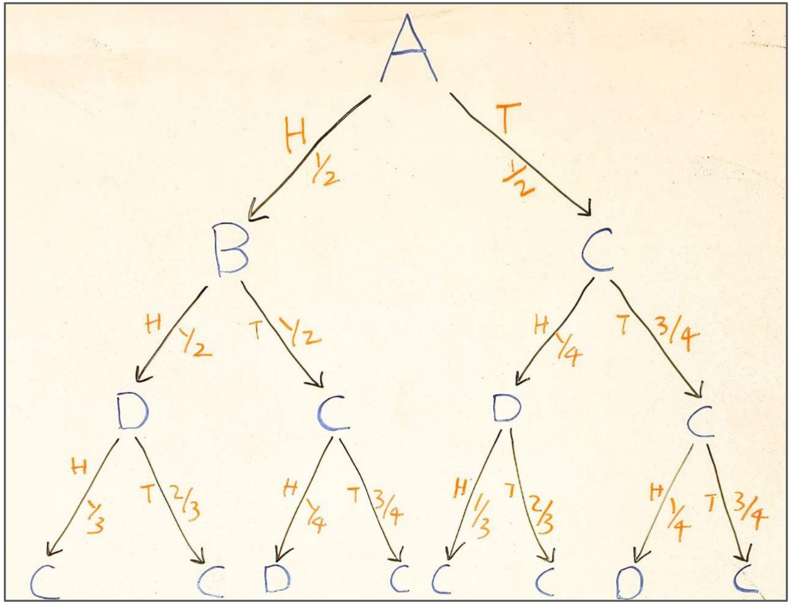




Coupled Coin :

What if we only have data of X ?

- Causal tree of X is more “complex” in the sense that it has more *causal states*



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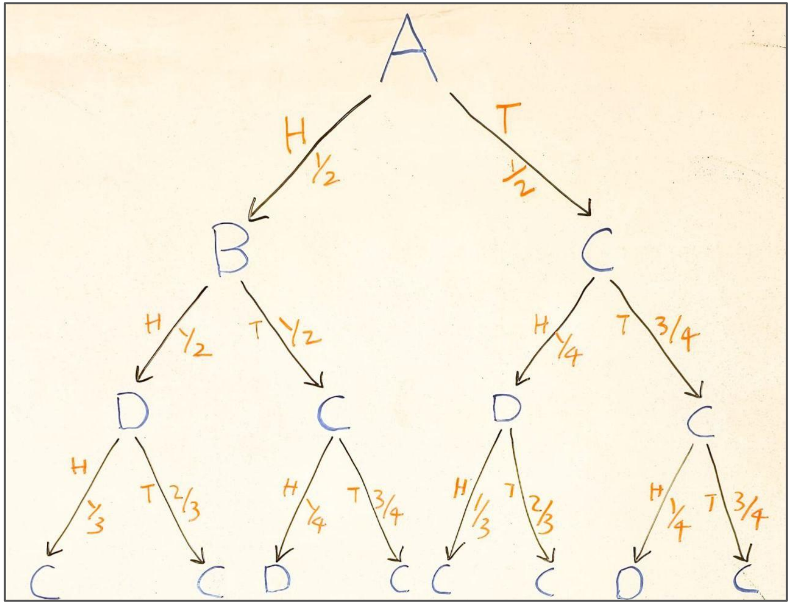
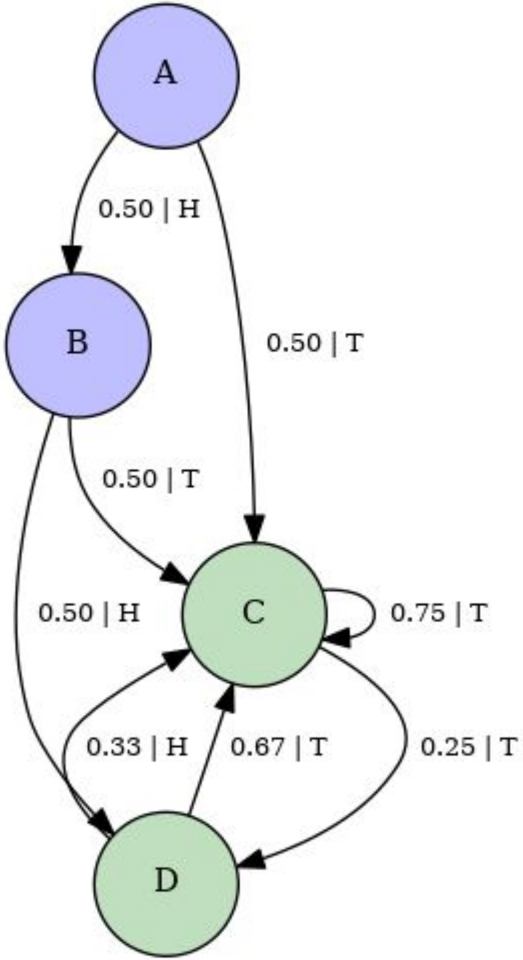


X :

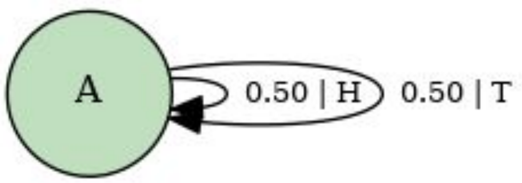


Coupled Coin:

What if we only have data of X ?

- Remember that Y influences X, so causal tree of Y is same as that of single fair coin

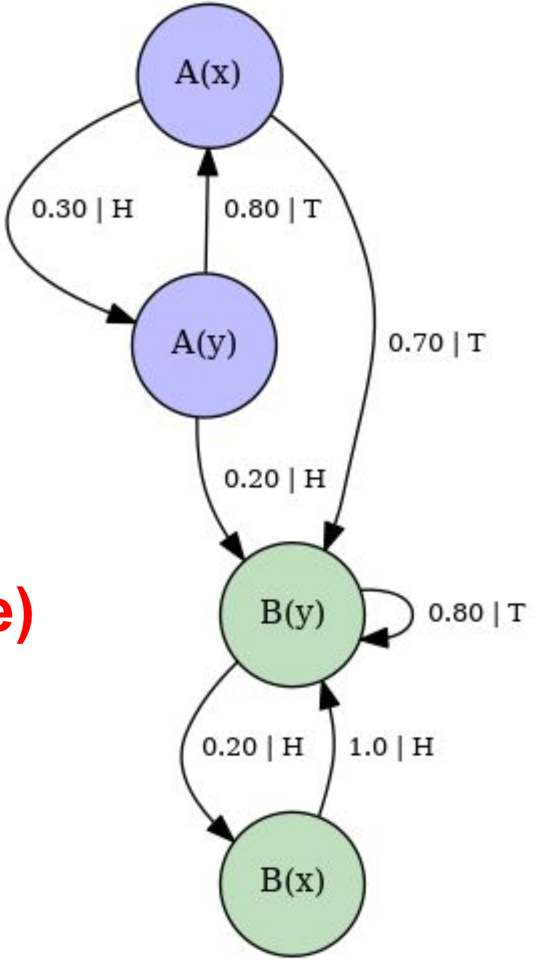
Y :



Coupled Coin: What we want?

Represent how causal states of Y and X influence each other

X & Y :
(Hypothetical case)



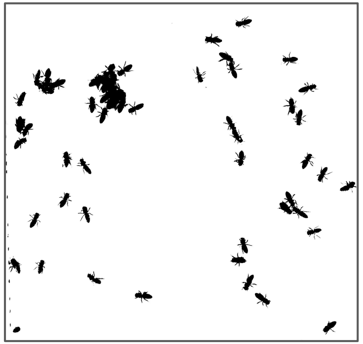
Applications: Question of interest

What are the minimal abstract machines underlying information processing in natural systems?

Applications: Study of collective behavior

- Model the mechanism behind the food exchange behavior in bees
- 2 time series describe the behavior as:

nonclustered (0)



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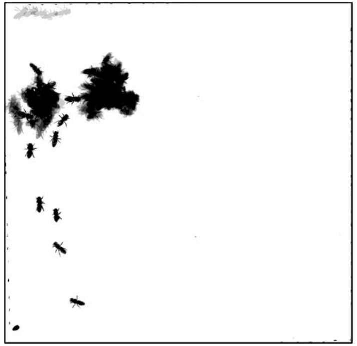
Clustered (1)



X: clustering behavior

nonscenting (0)

...

Scenting (1)

Y: scenting behavior

Redefine the Causal Equivalence Principle?

$$P(\overrightarrow{S}|\overleftarrow{S}_t) = P(\overrightarrow{S}|\overleftarrow{S}_{t'}) \iff \overleftarrow{S}_t \sim \overleftarrow{S}_{t'}$$

- Estimating higher order probability distributions from data - computationally expensive and requires long time series
- Can we use ***dynamical complexity*** instead of conditional probabilities?

$$DC(\overrightarrow{S}|\overleftarrow{S}_t) = DC(\overrightarrow{S}|\overleftarrow{S}_{t'})$$

- Foresee a relationship between *DC* and no. of causal states!

Kathpalia, A., & Nagaraj, N. (2019). Data-based intervention approach for Complexity-Causality measure. *PeerJ Computer Science*, *5*, e196.

Thank you!

धन्यवाद !

با تشکر!

¡Gracias!