QUESTION 1: Circuits & Formulas

Prove that $y_1 = (x_1 \text{ AND } x_2)$ is equivalent to $(x_1 \vee \bar{y}_1) \wedge (x_2 \vee \bar{y}_1) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee y_1)$ in the Boolean circuit shown below.

Complete the following statements about the three clauses. You may use either natural language or logical statements according to the notation used above.

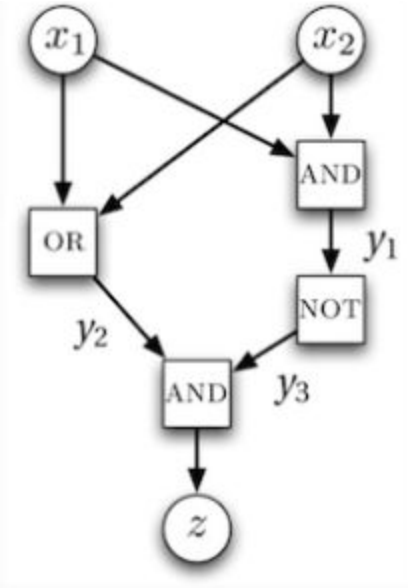
(a) Given than the three clauses are linked by logical AND statements, all three clauses must be ...

(b) If x_1 is false, y_1 must be...

(c) If x_2 is false, y_1 must be...

(d) If x_1 is true and x_2 is true, y_1 must be...

NOTE on notation: $\vee$ = logical OR; $\wedge$ = logical AND; $\bar{}$ = NOT (the statement is false)



QUESTION 2: Traveling Salesperson

A traveling salesperson needs to visit a series of cities connected by edges (roads). *D* is the distance of the **shortest** path through all of the vertices (cities) in the network. Which of these questions is in NP? Provide a brief explanation of your reasoning, eg. "It is easy to check if the solution is true by..."

Is *D* less than 10,000 miles?

Is *D* more than 8,000 miles?

Is *D* exactly 9,219 miles?

QUESTION 3: Complexity Hierarchy

Considering a cellular automata with a state *s* at time t_n, what is the complexity class that each of the following questions belongs to?

- What will the state be at t_{n+x}?
- Does *s* have a predecessor?
- On a lattice of size *n*, is *s* on a periodic orbit?
- On a lattice of infinite size, will *s* ever die out?

Your answers should indicate whether each question is in P, NP, PSPACE, or undecidable.

The questions correspond to the Circuits & Formulas (QUESTION 1), Traveling Salesperson (QUESTION 2) and Complexity Hierarchy (QUESTION 3) Quizzes, where you can find more information.

Download the video quizzes here.