

Unit 3 Homework: Information Theory and Computability

Algorithmic Information Dynamics: A Computational Approach to Causality and Living Systems From Networks to Cells

Hector Zenil, Narsis A. Kiani, Alyssa Adams

Course available at the Santa Fe Institute MOOC platform the Complexity Explorer at: <https://www.complexityexplorer.org/courses/63-algorithmic-information-dynamics-a-computational-approach-to-causality-and-living-systems-from-networks-to-cells>

Out[]= Tue 24 Jul 2018 17:00:48

■ Classical Information Theory

Question 1

What happens to the entropy of a random string when increasing the number of symbols, e.g. from binary to trinary? **Would the entropy increase or decrease?**

Answer 1

Question 2

What would happen if we used base 2 for our strings made of 0's and 1's? What would happen if we used base 3 for our strings made of 0's, 1's, and 2's? **Would the entropy increase or decrease?**

Answer 2

Question 3

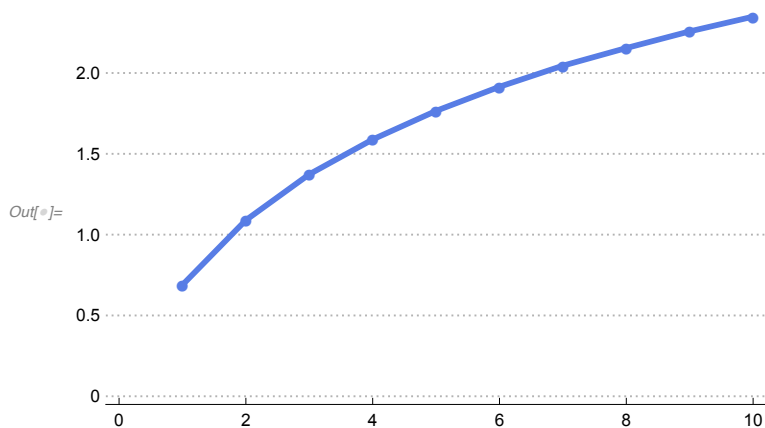
Lets keep e and investigate the behavior of entropy as a function of alphabet base.

Let's go all the way up to alphabet size 11.

```
In[ ]:= alphabetentropy =
  Table[Table[RandomChoice[Range[0, i], 100] // Entropy // N, 100] // Mean, {i, 1, 10}]
```

```
Out[ ]:= {0.689153, 1.08964, 1.37335, 1.58891,
  1.76578, 1.91495, 2.04536, 2.1545, 2.25763, 2.3486}
```

```
In[ ]:= ListPlot[alphabetentropy, PlotTheme → "Business",
  Joined → True, PlotMarkers → Automatic]
```



Why do you think the curve starts to slow down?

Answer 3

Question 4

Let's say we tossed a coin 1000 times and we got about 50% heads and 50% tails. If we measured the entropy of the heads and tails outcome, we would expect the entropy to be very close to 1, since it is a random process. However, say we found that the coin was weighted, with the tails side being heavier. How would this change the way we see our result?

Answer 4

Question 5

Say you find a mysterious DNA sequence. It has lots of interesting patterns and redundancy. For fun, you look at the first half of the sequence and given what you see, you are able to accurately predict the second half. Good job! Would this sequence have a higher or lower Shannon entropy? Does this mean the sequence is noisy?

Answer 5

■ Computability

Question 1

Is a universal Turing machine with 1 tape computationally more powerful than a universal Turing machine with 3 tapes? What about an infinite number of tapes?

Answer 1

Question 2

Write a transition table of rules for a Turing machine with only 2 states that always halts for any input.

Answer 2

Question 3

Describe what the following Turing machine will do, based on what you can see from its following rule transition table:

symbol, state -> new symbol, new state, head movement (1 is right, -1 is left)

0	1	-> 1	1	1
1	1	-> 0	1	1

Answer 3

Question 4

What the above Turing machine would do to the string “1010101”?

Answer 4

It would reverse it:

```
In[ ]:= RulePlot[TuringMachine[rules],
  {1, {1, 0, 1, 0, 1, 0, 1}}, 7, Mesh -> All, ImageSize -> Small]
```

Question 5

Write the previous Turing machine as a transition diagram.

Answer 5

Question 6

Is the above Turing machine universal?

Answer 6