Santa Fe Institute

Complex Systems Summer School

Prediction Markets

Rajiv Sethi

June 13, 2023

Overview

- What are prediction markets?
- Some current predictions for the 2024 Presidential Election
- Ecology of a complex system
- Coordination and manipulation
- Markets versus models
- Using markets to evaluate model performance

What is a Prediction Market?

- Mechanism for aggregating beliefs about likelihood of events
- Venue for the trading of securities with state contingent payoffs
- Sometimes called binary options or event futures
- Examples (real money, peer-to-peer): IEM, Intrade, PredictIt

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- Examples (real money, peer-to-peer): IEM, Intrade, PredictIt
- Contracts are liquid: traders can enter, exit, reverse positions
- Contracts have an expiration date, resolution based on public information
- Margin requirement is worst-case loss (exchange bears no risk)
- Forecasting performance can be evaluated based on Brier scores or calibration curves
- Historical accuracy competitive with poll aggregates and models

Current Quotes on PredictIt

Who will win the 2024 Republican presidential nomination?

Contract	Latest Yes Price	Best Offer		Best Offer
 Donald Trump	50¢ NC	51¢	Buy Yes / Buy No	50¢
 Ron DeSantis	33¢ 1¢↓	35¢	Buy Yes / Buy No	67¢
 Tim Scott	9¢ 1¢↑	9¢	Buy Yes / Buy No	92¢
 Glenn Youngkin	6¢ 2¢↑	6¢	Buy Yes / Buy No	95¢
 Nikki Haley	5¢ 1¢↓	6¢	Buy Yes / Buy No	95¢
 Mike Pence	4¢ NC	5¢	Buy Yes / Buy No	96¢
 Kristi Noem	2¢ NC	2¢	Buy Yes / Buy No	99¢

10 More Contracts

Who will win the 2024 Democratic presidential nomination?

Contract	Latest Yes Price	Best Offer		Best Offer
 Joe Biden	74¢ NC	74¢	Buy Yes / Buy No	27¢
 Gavin Newsom	15¢ 1¢↓	16¢	Buy Yes / Buy No	85¢
 Kamala Harris	7¢ NC	8¢	Buy Yes / Buy No	93¢
 Pete Buttigieg	4¢ NC	4¢	Buy Yes / Buy No	97¢
 Hillary Clinton	3¢ NC	4¢	Buy Yes / Buy No	97¢
 JB Pritzker	2¢ NC	2¢	Buy Yes / Buy No	99¢

6 More Contracts

Who will win the 2024 US presidential election?

	Contract	Latest Yes Price	Best Offer		Best Offer
	Joe Biden	45¢ NC	46¢	Buy Yes / Buy No	55¢
	Donald Trump	28¢ 1¢↓	29¢	Buy Yes / Buy No	72¢
	Ron DeSantis	27¢ NC	27¢	Buy Yes / Buy No	74¢
	Gavin Newsom	5¢ NC	6¢	Buy Yes / Buy No	95¢
	Kamala Harris	4¢ NC	5¢	Buy Yes / Buy No	96¢
	Pete Buttigieg	2¢ 1¢↓	3¢	Buy Yes / Buy No	98¢

Example

PredictIt 2024 GOP Nominee Contract:

- Each Contract pays $1 if referenced candidate wins, $0 otherwise
- If contract trades at $0.09, buyer posts $0.09, seller posts $0.91
- Buyer faces large gain with low probability, seller faces small but likely gain
- Trade driven by differences in beliefs and risk attitudes

Example

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Trading based on continuous double auction:

- All orders are limit orders (specify price and quantity)
- Non-marketable orders enter order book
- Marketable orders trade against a resting order

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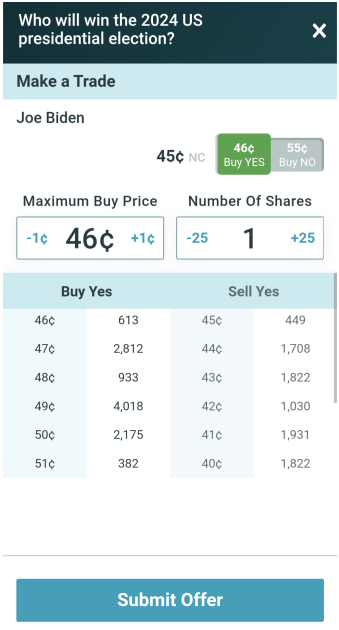
Can prices be interpreted as probabilities?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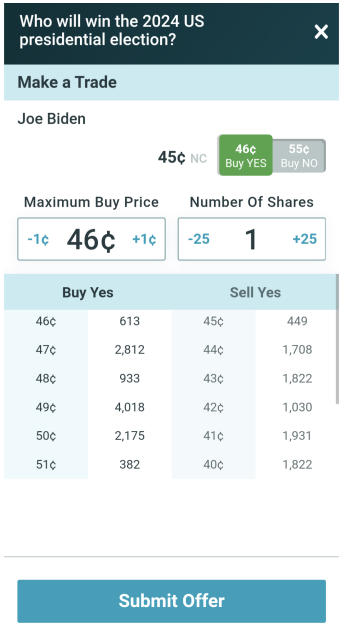
46¢ Buy YES
55¢ Buy NO
Maximum Buy Price
-1¢ 46¢ +1¢
Number Of Shares
-25 1 +25
Buy Yes
Sell Yes
46¢ 613 45¢ 449
47¢ 2,812 44¢ 1,708
48¢ 933 43¢ 1,822
49¢ 4,018 42¢ 1,030
50¢ 2,175 41¢ 1,931
51¢ 382 40¢ 1,822
Submit Offer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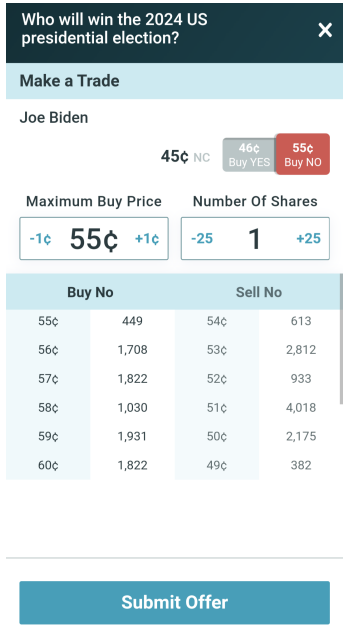
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Submit Offer

Market Ecology

- Prediction markets can serve as natural experiments to answer fundamental questions
- How does information get transmitted to prices?
- Price dynamics determined by ecology of trading strategies
- Need transaction level data to identify strategies

TRADING STRATEGIES AND MARKET MICROSTRUCTURE:

EVIDENCE FROM A PREDICTION MARKET*

David M. Rothschild *Rajiv Sethi*[‡]

TRADING STRATEGIES AND MARKET MICROSTRUCTURE: EVIDENCE FROM A PREDICTION MARKET*

David M. Rothschild *Rajiv Sethi*[†]

For each trade over existence of market (11/16/2010 to 11/7/2012)

- Time, Market, Price, Quantity, Buyer, Seller, Aggressor side

Data allows us to compute, for each account:

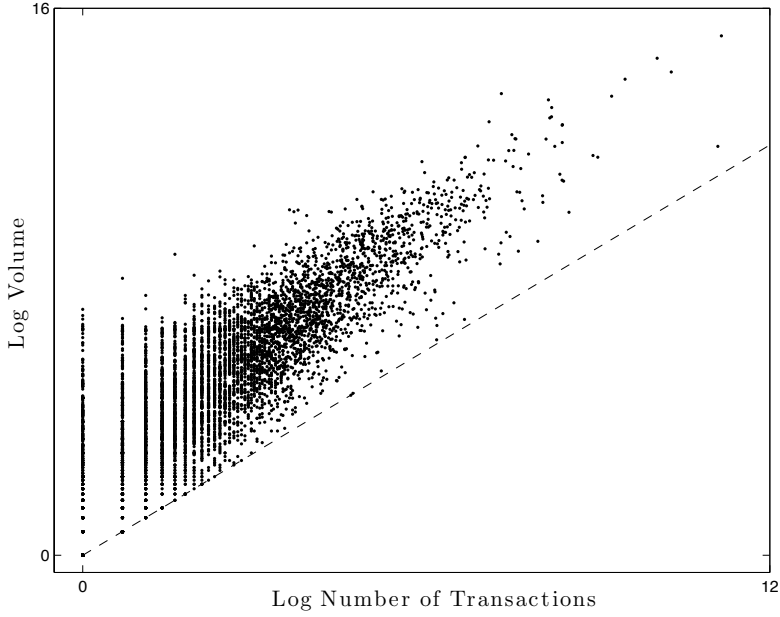
- Volume, Transactions, Aggression, Duration, Direction, Margin, Profit

Volume, Transactions, and Accounts

- 22 markets: Obama, Romney, Santorum, Gingrich, Palin, etc.
- 12.9 million contracts traded in 287,000 distinct transactions
- 7.6 million contracts in the major party nominees
- 6,300 unique trader accounts

Volume, Transactions, and Accounts

- 22 markets: Obama, Romney, Santorum, Gingrich, Palin, etc.
- 12.9 million contracts traded in 287,000 distinct transactions
- 7.6 million contracts in the major party nominees
- 6,300 unique trader accounts
- Average transaction size was 45, largest was 10,000
- Relatively small numbers of traders dominate volume and transactions
- 55% of traders had less than ten trades; 12% had just one
- Largest trader accounts for 15% of volume, 12% of trades
- One percent of traders account for 67% of volume
- One percent account for 60% of transactions


16
0
Log Volume
0
12
Log Number of Transactions

Classifying Strategies

- Aggression: proportion of orders that are marketable
- Holding Period: Median time between entry and exit of a position
- Duration: holding period relative to median time to expiration
- Margin: maximum amount at risk
- Profit: Net gain across all trades

Characteristics of Selected Traders

	Volume	Trades	Aggression	Direction	Holding	Duration	Margin	Profit
A	3,961,242	69,977	0.77	0.19	0	0.00	$9,877	$61,871
B	2,062,908	22,738	0.27	−1.00	520,428	1.00	$6,882,186	-$6,882,186
C	1,380,406	29,134	0.31	−0.49	2,491	0.00	$737	$11,921
D	321,818	1,207	0.83	0.92	51,470	1.00	$2,099,441	$867,059
E	174,712	4,340	0.79	−0.13	7	0.00	$415	$1,058
F	156,413	65,652	0.72	0.38	0	0.00	$1,375	$2,147
G	138,264	1,707	0.39	1.00	1,802,885	1.00	$535,018	$318,975
H	72,563	392	0.73	1.00	149,180	1.00	$479,896	$233,414
I	68,416	858	0.37	−1.00	664,333	1.00	$121,609	-$121,609
J	44,195	350	0.64	−1.00	152,360	1.00	$149,998	-$149,998

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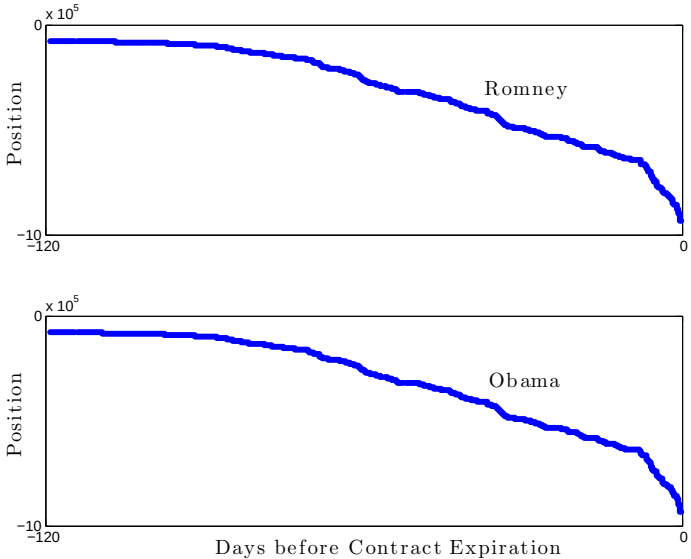
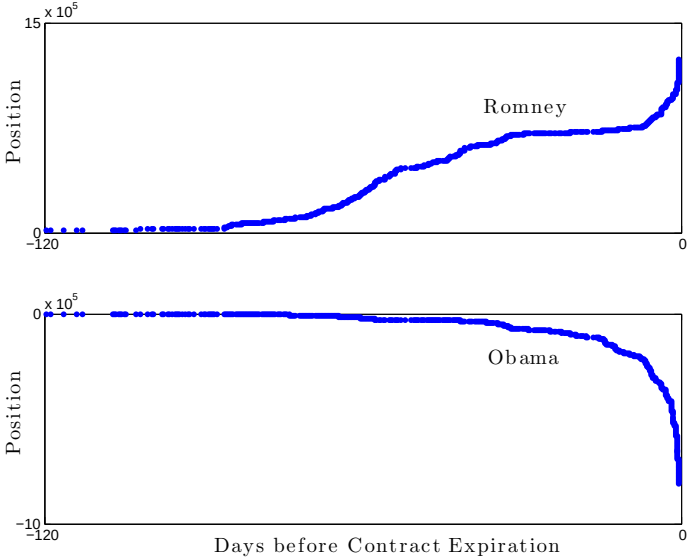
Trader A: Largest Volume and Most Transactions



Trader B: Largest Directional Exposure and Loss

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WASHINGTON WIRE
One Big Trader Lost Millions Betting on Romney, Study Finds


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NETNET
Romney loss cost trader up to $7 million: Study
Jeff Cox | @JeffCoxCNBCcom
Published 11:40 AM ET Wed, 25 Sept 2013

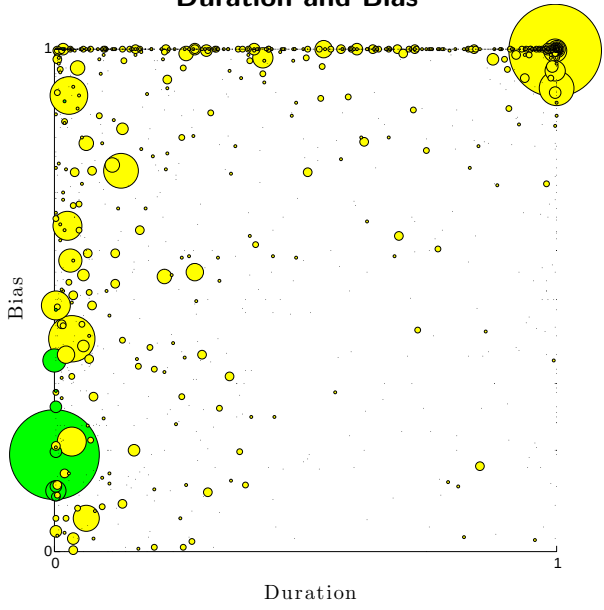
Taxonomy of Trading Strategies

- Arbitrage: median holding period less than 10 minutes (A, E, and F)
- Unidirectional: bias equal to 1, non-arb (G, H, I, and J)
- Extreme Bias: bias above 0.9 but below 1, non-arb (B and D)
- High Bias: bias between 0.5 and 0.9, non-arb
- Moderate Bias: bias between 0.25 and 0.5, non-arb (C)
- Low Bias: bias less than 0.25, non-arb

A Taxonomy of Trading Strategies

Strategy	Traders	%	Volume	%	Aggression	Duration	Bias
Unidirectional	5,118	87%	4,901,262	32%	0.65	0.75	1.00
Extreme Bias	136	2%	3,987,006	26%	0.38	0.65	0.97
High Bias	272	5%	1,699,355	11%	0.42	0.22	0.71
Moderate Bias	173	3%	1,293,289	9%	0.41	0.06	0.43
Low Bias	167	3%	926,702	6%	0.40	0.10	0.13
Arbitrage	40	1%	2,368,380	16%	0.73	0.00	0.20
Total	5,906	100%	15,175,994	100%			

Duration and Bias



Coordination and Manipulation

- Can market prices affect probability of referenced event?
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The Prediction Market Paradox

- Incentives to manipulate forecasts believed to be unbiased
- But not forecasts believed to be manipulated
- Forecasting accuracy is self-limiting
- Is there evidence of attempted manipulation?

Election Day Romney Order Book (3:30 pm)

BID		ASK	
Qty	Price	Price	Qty
678	30.3	30.6	1
10	30.1	30.7	15
9999	30.0	30.8	119
8505	29.9	30.9	223
4890	29.2	31.0	839
6900	29.0	31.1	202
4999	28.5	31.2	800
10000	28.0	32.0	101
647	27.8	32.7	299
2318	27.5	32.8	300
598	27.1	32.9	1654
1338	27.0	33.0	50
1	26.9	33.3	456
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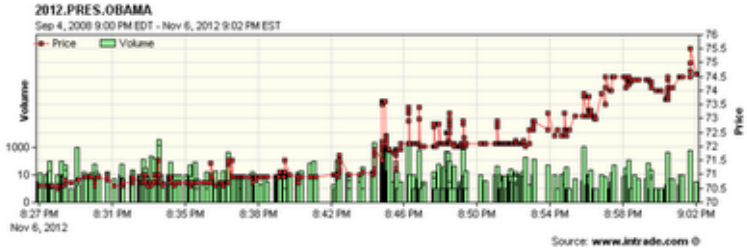
Floor and Ceiling



Rajiv Sethi
@rajivatbarnard

Romney order book on Intrade is amazing, someone with deep pockets trying to hold price above 30 @justinwolfers pic.twitter.com/EHvm1DGl

Reply Delete Favorite More

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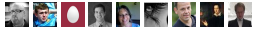
16 RETWEETS



Justin Wolfers
@justinwolfers

Following

Wow RT @rajivatbarnard Romney order book on Intrade is amazing, someone with deep pockets trying to hold price above 30 pic.twitter.com/df2HIlia

Reply Retweet Favorite More

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65 RETWEETS 8 FAVORITES

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Was There Manipulation?

- Evidence suggestive but not decisive
- Consistent belief that market was undervaluing Romney?
- Financial motives possible but hard to identify
- Campaign related motives seem more plausible

Prediction Market Performance

- Wishful thinking and manipulation incentives should lower performance
- But prediction market accuracy has been consistently high
- What accounts for this? Shouldn't models produce better forecasts?
- How does market performance compare with model performance?

Models, Markets, and the Forecasting of Elections

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JULIE SEAGER and EMILY CAI, Department of Economics, Barnard College, Columbia University, USA
DANIEL M. BENJAMIN and FRED MORSTATTER, USC Information Sciences Institute, USA

Overview

Two approaches to forecasting: models and markets

- Models (typically) backward-looking, limited set of variables, sluggish adjustment
- Markets forward-looking, use arbitrary information sources, rapid adjustment

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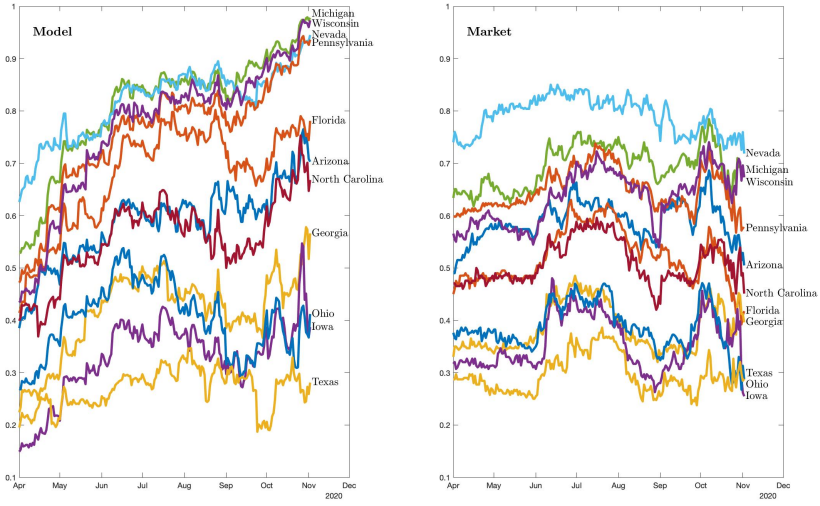
This paper

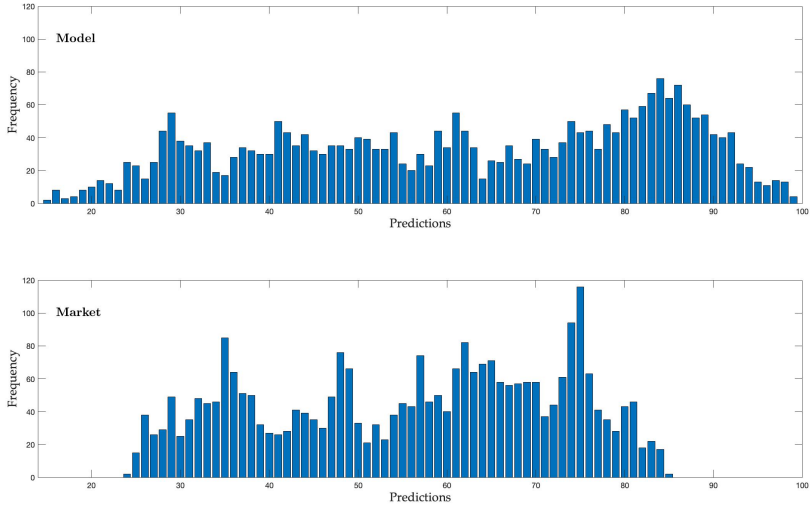
- Examine daily forecasts from Economist model and PredictIt exchage
- Data for 13 battleground states over 216 consecutive days leading to election
- Comparative performance evaluation, value of integration, method of hybridization
- Models can incorporate prices, markets can incorporate model-based beliefs
- Trading bot updates beliefs and portfolios daily, given endowment and risk preferences
- Bot profitability can be used for comparative model evaluation
- Check for robustness by computing profitability under different election outcomes

Data

- Daily closing prices and daily model forecasts for thirteen battleground states
- Arizona, Florida, Georgia, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Texas, Wisconsin
- Significant disagreement between model and market in many states
- Model spans larger range of probabilities
- Model has greater movement across this range for individual states



Probabilities of a Democratic Victory in Selected States



Frequency Distributions for Model and Market Predictions

Performance

- Let p_{it} denote the probability assigned in period t to a Democratic victory in state i
- Brier score for period t, state i forecast

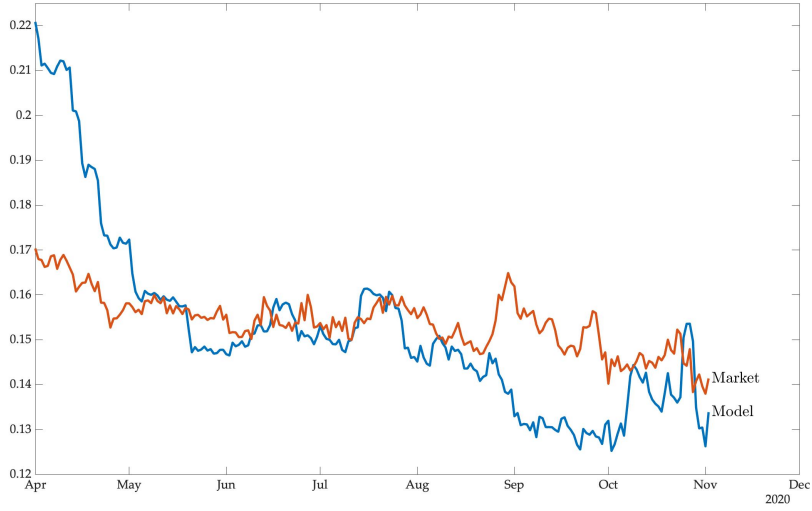
$$s_{it} = (p_{it} - r_i)^2$$

where $r_i = 1$ if state i resolved in favor of Democratic nominee, $r_i = 0$ otherwise

- We have $T = 216$ periods (consecutive days) and $n = 13$ states
- Averaging across states, obtain time series of average Brier score

$$\bar{s}_t = \frac{1}{n} \sum_{i=1}^{n} (p_{it} - r_i)^2$$

that is computed for models and markets separately



Mean Brier Scores for the Model and Market over Time

Overall Performance

Averaging across time as well as states, obtain a scalar measure of overall performance:

$$\bar{s} = \frac{1}{nT} \sum_{t=1}^{T} \sum_{i=1}^{n} (p_{it} - r_i)^2$$

where $T = 216$ and $n = 13$

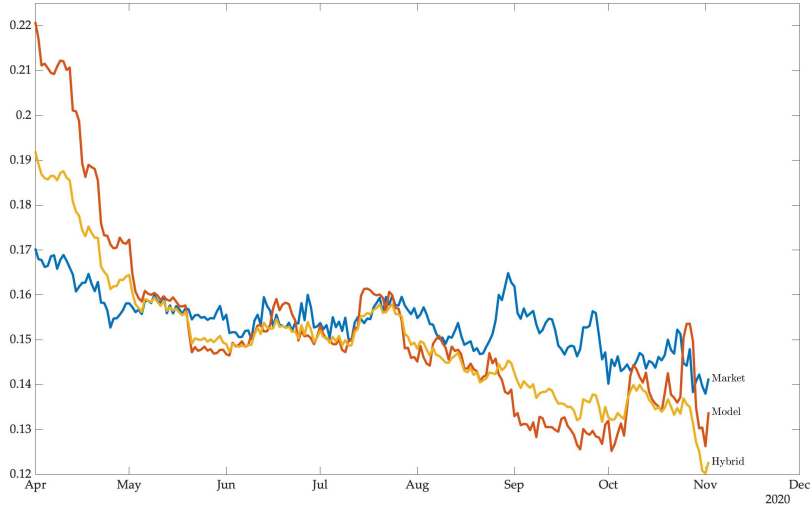
Virtually identical average forecasting performance across mechanisms:

$$\begin{aligned} \bar{s}^{model} &= 0.1523 \\ \bar{s}^{market} &= 0.1539 \end{aligned}$$

However, this conceals significant differences in performance over time

Averaging Forecasts

- For each state-date pair, compute simple average probability of Democratic victory
- For any given state-date pair, average cannot get better score than both components
- But this is no longer true when we average across states and/or dates
- On 87 (of 216) days average performs better than both model and market
- Includes the 26 days leading up to the election



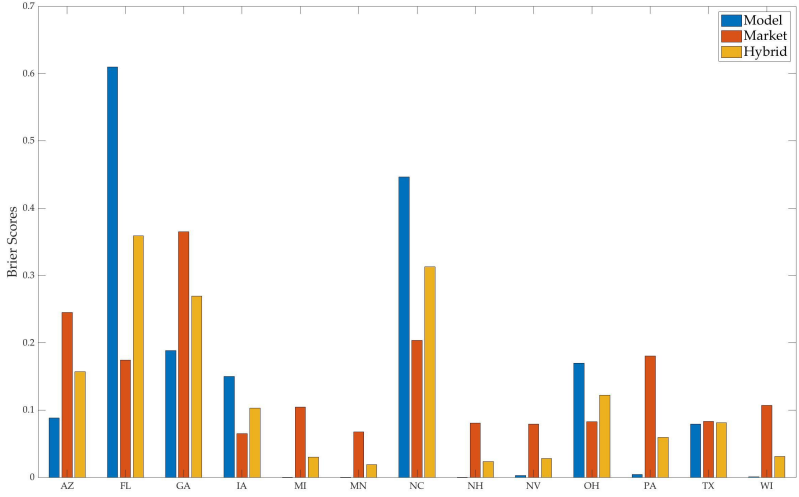
Mean Brier Scores for the Model, Market, and a Simple Average over Time

Averaging Forecasts

- Across all periods and states, average Brier score is

$$\bar{s}^{hybrid} = 0.1499$$

  so simple average of the two component forecasts beats both market and model
- Model and market made errors in different ways and for different states
- Model confidently wrong in FL/NC, market too uncertain in MN/NH
- Average avoids the most egregious errors
- Scores for last day: 0.1414 for market, 0.1339 for model, and 0.1228 for average



Brier Scores for Market, Model, and Hybrid Forecasts on November 2

Hybrid Prediction Market

- Algorithmic trading common in financial markets, low latency can be highly rewarding
- Insert into market a trading bot that acts as if it believes the model forecast
- Bot endowed with a budget and preferences that exhibit risk aversion
- Bot posts orders based on model-based beliefs, preferences, and current portfolio
- Orders trade immediately or enter order book, providing liquidity to market
- Cash and asset holdings of bot evolve over time, affecting the prices and sizes of orders
- Parameters can be tuned in experimental settings to examine effects on accuracy

Notation

Model

- Model generates distribution over outcomes in m jurisdictions, n candidates in each
- $\mathbf{S}$ denotes outcome realization, $s_{ij} = 1$ indicates candidate i wins jurisdiction j
- Ω is set of possible outcomes, $p : \Omega \to [0, 1]$ is model forecast
- With two candidates and thirteen states $|\Omega| = 2^{13} = 8192$

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Market

- For each jurisdiction, prediction market lists n contracts, one for each candidate
- Each contract has a unique price at which it can be bought/sold (ignore bid-ask spread)
- $\mathbf{Q}$ denotes prices, $q_{ij} \in [0, 1]$ is price of contract that pays if i wins in j

Trading

- Bot portfolio is $(y, \mathbf{Z})$, where y is cash and $\mathbf{Z}$ is $n \times m$ matrix of contract holdings
- If outcome is $s \in \Omega$, terminal wealth resulting from portfolio $(y, \mathbf{Z})$ is

$$w = y + \sum_{j \in M} \mathbf{s}_j' \mathbf{z}_j,$$

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- Given starting portfolio, beliefs, preferences, and prices, trades $\mathbf{X}$ chosen to maximize

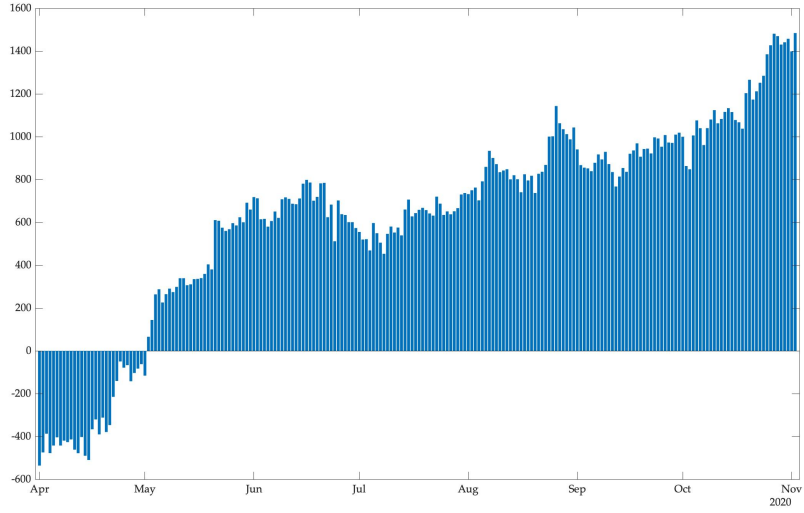
$$E(u) = \sum_{S \in \Omega} p(S) u \left(y + \sum_{j \in M} \left(\mathbf{s}_j' (\mathbf{z}_j + \mathbf{x}_j) - \mathbf{q}_j' \mathbf{x}_j \right) \right)$$

Implementation

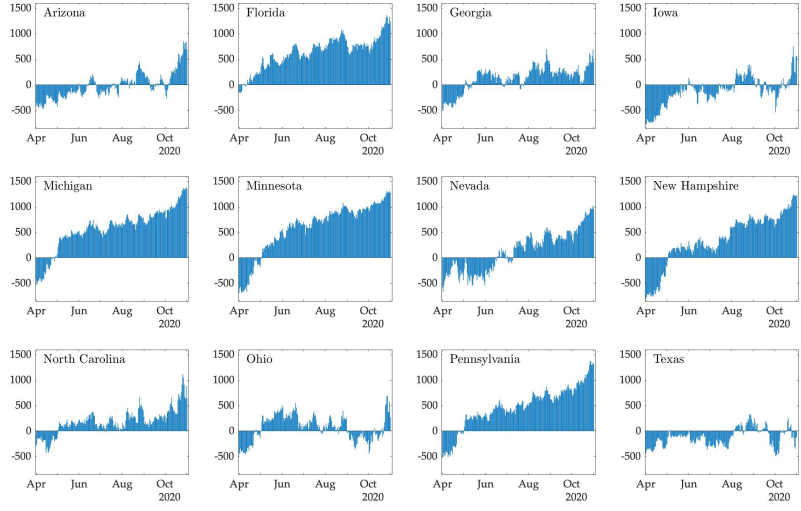
- Consider two candidates, one market
- Preferences exhibit constant relative risk aversion (CRRA):

$$u(w) = \begin{cases} \frac{1}{1-\rho} w^{1-\rho}, & \text{if } \rho \geq 0,\ \rho \neq 1 \\ \log(w), & \text{if } \rho = 1 \end{cases}$$

  where $\rho = 0$ is risk-neutrality, higher ρ involves greater risk aversion
- Initial portfolio $(y, z) = (1000, 0)$
- How would portfolios have evolved in the case of Wisconsin?



Evolution of Contract Holdings for Wisconsin



Bot Contract Holdings for 12 battleground States

State	Cash	Contracts	Value	Payoff	Profit	Return
Arizona	$566.45	748.55	$944.35	$1,314.99	$314.99	31%
Florida	$339.96	1351.73	$904.28	$339.96	–$660.04	–66%
Georgia	$743.54	735.23	$1,034.72	$1,478.76	$478.76	48%
Iowa	$855.20	722.99	$1,039.50	$855.20	–$144.80	–14%
Michigan	$68.29	1383.63	$1,004.27	$1,451.92	$451.92	45%
Minnesota	$54.98	1305.55	$1,021.09	$1,360.53	$360.53	36%
Nevada	$190.84	1094.15	$976.93	$1,284.99	$284.99	28%
New Hampshire	$72.74	1274.46	$984.85	$1,347.20	$347.20	35%
North Carolina	$607.15	880.03	$1,004.03	$607.15	–$392.85	–39%
Ohio	$904.82	669.22	$1,096.97	$904.82	–$95.18	–10%
Pennsylvania	$138.91	1359.79	$921.43	$1,498.70	$498.70	50%
Texas	$1,027.13	–36.95	$1,016.47	$1,027.13	$27.13	3%
Wisconsin	$89.92	1484.09	$1,088.83	$1,574.01	$574.01	57%
Total			$13,037.72	$15,045.36	$2,045.36	16%

Terminal portfolios, payoffs, and profits in battleground states.

Robustness

- How would model have performed if one or more close states had been decided differently?
- Closest states: GA (0.24 percent), AZ (0.31 percent) and WI (0.62 percent)
- What if one or more of these had been decided differently?

Flipped State(s)	Payoff	Profit	Rate
Georgia	$14,310.14	$1,310.14	10.08%
Arizona	$14,296.82	$1,296.82	9.98%
Wisconsin	$13,561.27	$561.27	4.32%
Georgia, Arizona	$13,561.60	$561.60	4.32%
Georgia, Wisconsin	$12,826.05	-$173.95	–1.34%
Arizona, Wisconsin	$12,812.73	-$187.27	–1.44%
Georgia, Arizona, Wisconsin	$12,077.51	-$922.49	–7.10%

Hypothetical payoffs and profits if the closest states had been decided differently.

Conclusions

- Overall performance differences negligible, but interesting patterns in time series
- Market does better early, worse late
- Possible distortions due to massive inflow of funds, increase in volume
- Simple average beats both components overall and at the end of the period
- Suggest value of hybridization; we propose a hybrid prediction market
- Based on virtual trader with tunable budget and preferences; model-based beliefs
- Profitability test can be used for comparative evaluation of model performance

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