

# Santa Fe Complex Systems Summer School

## MERITOCRACY AND REPRESENTATION

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## **Meritocracy and Representation**

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## Meritocracy in the Face of Group Inequality

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## The Meaning of Meritocracy

Standard conception of meritocracy in selection practices:

- (Strictly) increasing in measures of past performance (monotonicity)
- Contingent only on productive traits (group-blindness)

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But past performance is a garbled signal and group-membership can be informative

- Under what conditions will monotonic and group-blind selection maximize performance?
- What is the structure of performance-maximizing policies more generally?
- Can a disadvantaged group be overrepresented if performance is the only goal?
- What are the effects of allowing for underreporting or underinvestment?
- What are the effects of allowing for commitment?

We do not...

... argue that meritocracy is a desirable standard

... or that it is approximated in reality

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John Adams: any elite, once formed, would entrench itself indefinitely: "Aristocracy, like Waterfowl, dives for Ages and then rises with brighter Plumage. It is a subtle Venom that diffuses itself unseen, over Oceans and Continents, and tryumphs over time... it is a Phoenix that rises again out of its own Ashes."

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These findings make sense if the "meritocratic" benchmark is not performance-maximizing

#### **Profiling versus Preferential Treatment**

A student who has, in some way, experienced hardship may underperform on achievement tests relative to his or her capability. By taking account of such empirically grounded differences across demographic groups, a district may be better able to determine which students are most suited to admission to the gifted program... While this profiling based on differences in distributions across racial groups is beneficial to minority students, it is not preferential treatment.

Cestau, Epple, and Sieg (2017)

## Overview

Suppose selection is based only on expected performance and

- Individuals differ along two dimensions: ability and resources
- Both ability and resources affect qualifications or training
- Training is observable (via a score) but ability and resources are not
- Performance depends on both ability and training
- So ability affects performance both directly and indirectly
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What is the optimal selection policy?

What if individuals can underinvest in training relative to potential?

What if commitment to a selection policy is possible?

## A Discrete Case

Two groups, populations shares  $s_1$  and  $s_2$ 

Ability levels  $\{a_I, a_h\}$ , common ability distribution F(a)

**Resource** levels  $\{r_i, r_h\}$ , proportion with high resource in group *i* is  $q_i$ 

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Benchmark (no underinvestment): four training levels  $t_{ij} = \tau(a_i, r_j)$  with

 $t_{II} < t_{Ih}, t_{hI} < t_{hh}$ 

## Monotonicity

Two types of monotonicity:

- inference monotonicity higher levels of training signal higher ability
- performance monotonicity expected performance increasing in training

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If  $t_{lh} > t_{hl}$ , inference monotonicity fails and performance monotonicity

- holds if  $\phi(a_l, t_{lh}) > \phi(a_h, t_{hl})$ ;
- fails if  $\phi(a_l, t_{lh}) < \phi(a_h, t_{hl})$

If performance monotonicity fails, selection is non-monotonic in training

## An Example

Suppose that  $t_{lh} > t_{hl}$  and  $\phi(a_h, t_{hl}) > \phi(a_l, t_{lh})$ , so performance monotonicity fails to hold

Population shares  $s_1 = s_2 = 0.5$ 

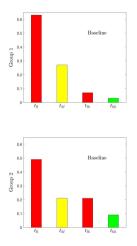
Ability distribution (common):  $F(a_I) = 0.7$ 

Resource Distributions:  $q_1 = 0.1$ ,  $q_2 = 0.3$ 

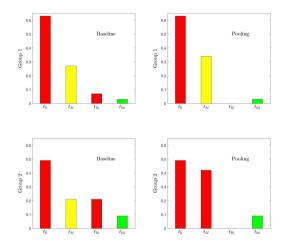
Elite Capacity k = 10%

Performance-maximizing selection policy accepts all  $t_{hh}$  scores (6% of population)

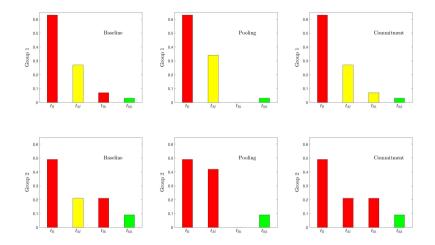
But how are the remainder selected?



Equal size groups, 30% high ability, 10% and 30% high resources, 10% elite capacity



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## **Comparing Regimes**

Baseline:

- Selection is non-monotonic but (possibly) group-blind
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- Disadvantaged group favored under performance-maximization (55% of those selected)
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Pooling:

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- Selection effect overwhelms disadvantage

Commitment:

- Same individuals selected as under pooling
- No underinvestment, so average performance is higher

How general is this?

## Notation

Let  $\pi_i(t)$  denote selection probability conditional on training t for group i

Policy is (strictly) monotonic if  $\pi_i(t) > 0$  implies  $\pi_i(t') = 1$  for all t' > t

Policy is group-blind if  $\pi_1(t) = \pi_2(t)$  at all t

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Define  $k_1$  as proportion of population with high ability and resources:

$$k_1 = (1 - F(a_I))(s_1q_1 + s_2q_2)$$

and  $k_2$  as proportion with high ability or resources

$$k_2 = 1 - F(a_I)(s_1(1-q_1) + s_2(1-q_2))$$

#### **Conditions for Monotonicity and Group-Blindness**

If performance monotonicity holds, then there exists a monotonic and group-blind equilibrium

If performance monotonicity fails to hold, there is monotonic and group-blind equilibrium if and only if  $k < k_1$  or  $k > k_2$ 

Under monotonic selection, no incentive to underinvest or underreport training But no monotonic and group-blind equilibrium for intermediate elite capacity

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Under monotonic selection, no incentive to underinvest or underreport training But no monotonic and group-blind equilibrium for intermediate elite capacity What does equilibrium lock like if  $k \in (k - k_0)^2$ 

What does equilibrium look like if  $k \in (k_1, k_2)$ ?

## Intermediate Elite Capacity

Define  $\lambda_i$  as proportion with high ability in group *i* among those with  $t \in \{t_{hl}, t_{lh}\}$ 

$$\lambda_{i} = \frac{(1 - q_{i})(1 - F(a_{l}))}{(1 - q_{i})(1 - F(a_{l})) + q_{i}F(a_{l})}$$

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Suppose performance monotonicity fails, so  $t_{hl} < t_{lh}$  and

 $\phi(a_h, t_{hI}) > \phi(a_I, t_{Ih})$ 

Then there exists  $\mu \in (0, 1)$  such that

$$\mu\phi(a_h, t_{hl}) + (1-\mu)\phi(a_l, t_{hl}) = \phi(a_l, t_{lh})$$

This is a pool composition such that performance is equalized across training levels

 $\lambda_1 > \lambda_2 > \mu$ 

	$\pi(t_{II})$	$\pi(t_{hl})$	$\pi(t_{hl})$	$\pi(t_{hh})$
Group 1	0	+	0	1
Group 2	0	0	0	1

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All underinvest in both groups, disadvantaged group favored

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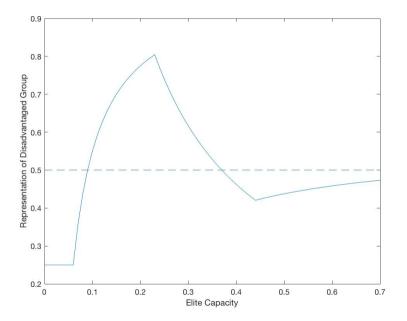
	$\pi(t_{II})$	$\pi(t_{hl})$	$\pi(t_{hl})$	$\pi(t_{hh})$
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All underinvest in group 1, some in group 2, disadvantaged group favored

$$\mu > \lambda_1 > \lambda_2$$

	$\pi(t_{II})$	$\pi(t_{hl})$	$\pi(t_{hl})$	$\pi(t_{hh})$
Group 1	0	+	+	1
Group 2	0	+	+	1

Some underinvest in both groups, selection is group-blind and weakly monotonic



## Extensions

## Commitment

- Underinvestment is wasteful since training is productive
- Can implement the same outcomes with commitment to weakly monotonic selection

Robustness

- Extensions to multiple ability and resource levels straightforward
- Qualitative results hold even with continuous distributions

## **Continuous Ability Distributions**

Common ability distribution F(a), density f(a) > 0, support [0, 1]

Benchmark case (no underinvestment): training  $t = \tau(a, r)$ 

Define  $\overline{t} = \tau(1, r_h)$  and  $t^* = \tau(1, r_l)$ ; note  $t^* < \overline{t}$ 

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Selection rule (deterministic): possibly group-contingent sets of training levels  $T_1$  and  $T_2$  to maximize expected performance subject to capacity constraint

## **Ability and Training**

Given any t, define  $\alpha_l(t)$  and  $\alpha_h(t)$  implicitly by

$$t=\tau(\alpha_I,r_I)$$

and

$$t=\tau(\alpha_h,r_h)$$

Note:  $\alpha_I(t) > \alpha_h(t)$ , and both  $\alpha_h$  and  $\alpha_I$  are strictly increasing in t

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Note:  $\alpha_I(t) > \alpha_h(t)$ , and both  $\alpha_h$  and  $\alpha_I$  are strictly increasing in tGiven any set of training levels T, define  $A_h(T)$  and  $A_I(T)$  implicitly by

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An allocation is group-blind if  $T_1 = T_2$ 

# **Expected Performance**

Let  $\gamma_i(t)$  denote likelihood that individual in group *i* with training *t* has high resource access

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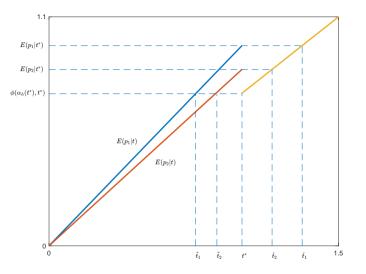
$$E(p_i|t) = \gamma_i(t)\phi(\alpha_h(t), t) + (1 - \gamma_i(t))\phi(\alpha_l(t), t)$$

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$$E(p_i|t) = \gamma_i(t)\phi(\alpha_h(t), t) + (1 - \gamma_i(t))\phi(\alpha_l(t), t)$$

- For  $t > t^*$  only high resource individuals are observed, so performance is increasing in t and identical across groups
- For  $t \le t^*$  both resource levels are pooled, and there is some  $t \le t^*$  with higher expected performance than some  $t' > t^*$



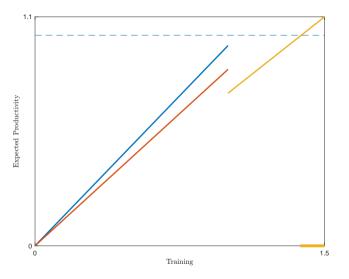
Uniform Ability and Linear Training and Performance Functions

# Regimes

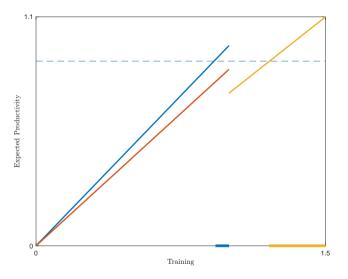
Performance-maximizing allocation must lie in one of four regimes

- For small k allocation is monotonic and group-blind
- For larger k disadvantaged group faces non-monotonic selection
- For even larger k, both groups face nonmonotonic selection
- For still larger k allocation is monotonic but not group-blind

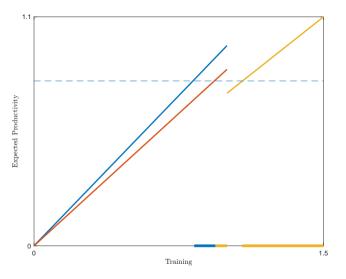
Standard for disadvantaged group less restrictive except in first regime



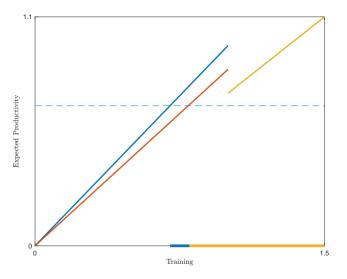
Regime 1: Monotonic and Group-Blind Selection



Regime 2: Nonmonotonic Selection for Disadvantaged Group



Regime 3: Nonmonotonic Selection for Both Groups



Regime 4: Monotonic (but not Group-Blind) Selection

## Underrepresentation

Given any performance-maximizing allocation  $(T_1, T_2)$ , let  $\rho_i$  be defined

$$ho_i = rac{1}{k} \left( q_i \int_{\mathcal{A}_h(\mathcal{T}_i)} d\mathcal{F}(\mathbf{a}) + (1-q_i) \int_{\mathcal{A}_l(\mathcal{T}_i)} d\mathcal{F}(\mathbf{a}) 
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Measure of the degree to which group i is underrepresented

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Measure of the degree to which group i is underrepresented

Note:  $\rho_1 = \rho_2 = 1$  if and only if the allocation is group egalitarian

Disadvantaged group is underrepresented if  $ho_1 < 1 < 
ho_2$ 

Clearly disadvantaged underrepresented at pseudomeritocratic allocation

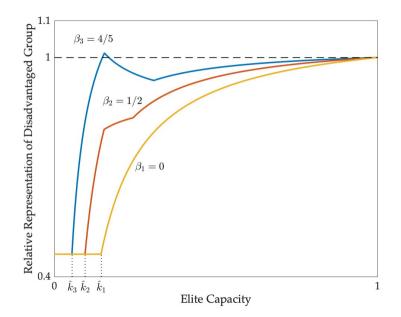
But is this true at all allocations?

# Example

- Groups are of identical size
- Resource access:  $q_1 = 1/5$ ,  $q_2 = 2/3$
- Resource levels:  $r_l = 1$ ,  $r_h = 3/2$
- Training t = ar
- Uniformly distributed ability

Consider parametric specification for performance:

$$p = \phi(a, t) = \beta a + (1 - \beta)t.$$



# Conclusions

Unconstrained allocations motivated by expected performance can exhibit

- Non-monotonic selection within groups
- Non-uniform selection across groups
- Less restrictive criteria for disadvantaged groups
- Overrepresentation of disadvantaged under certain conditions
- Imposing monotonicity and group-blindness can result in welfare losses
- Even greater than requiring statistical mirroring

Capacity constraint and importance of ability are key parameters

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Usual framing of the problem of affirmative action as a trade-off between performance and representativeness is misleading

Resource differences make identity informative about ability and performance